Worksheet 4: Intro to Derivatives

Instructions:

1. In this exercise you will construct one definition of derivative of \( f(x) \), using the graph above.
   (a) Determine the coordinates of the two bold points and fill in the blanks.
   (b) Find the vertical distance between the two points and label it above at (b).
   (c) Write an expression for the slope of the secant line, \( S \).
      \[
      \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(x+h)}{h} = \frac{f(x+h) - f(x)}{h}
      \]
      
      (d) In the limit as \( h \) approaches zero, what does the line \( S \) become?
      The tangent line at \((x, f(x))\)
      
      (e) Write an expression for the slope of the tangent line at \( x \). (Hint: use part (c) and take a limit.)
      \[
      \lim_{h \to 0} (\text{slope of secant line}) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
      \]
      
      This is the derivative of \( f \) at \( x \)!

2. The function plotted above is actually \( f(x) = 40x - 16x^2 \).
   (a) Find \( f'(x) \) using the definition of derivative you derived in 1(e).
   \[
   f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[40(x+h) - 16(x+h)^2] - [40x - 16x^2]}{h}
   = \lim_{h \to 0} \frac{40h - 32xh - 16h^2}{h}
   = \lim_{h \to 0} (40 - 32x - 16h)
   \]
   
   (b) Calculate the derivative at \( x = 2 \).
   \[
   f'(2) = 40 - 32x \quad \text{so} \quad f'(2) = 40 - 32(2) = -24
   \]
   
   (c) If \( f(x) \) is a function of distance versus time, what does \( f'(2) \) represent?
   -the instantaneous rate of change of distance with respect to time
   
   (d) On Quiz 1, you estimated the instantaneous velocity at \( t=2 \) of a ball whose height at time \( t \) is given by \( f(t) = 40t - 16t^2 \). Compare your quiz answer to \( f'(2) \).
   -they agree!