Math 1271
Fall, 2004
FINAL EXAM

Name (Print)______________________________

Signature______________________________

T.A. Instructor____________________ Discussion Section________ I.D.#________

READ AND FOLLOW THESE INSTRUCTIONS

This booklet contains 13 pages, including this cover page. Check to see if any are missing. <u>PRINT</u> on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. [redacted]. Do your work in the blank spaces and back of pages of this booklet. Show all your work.

There are 12 machine-graded problems worth 8 points each and 6 hand-graded problems worth 104 points together for a total of 200 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-12):
You <u>MUST</u> use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instrucitons you receive. **DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET**. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 13-18):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam. Either the student or the School of Mathematics may for any reason request a regrading of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. *Therefore work and answers must be clearly shown on the test booklet.*

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing **DOWN**. Have your ID card in your hand when turning in your exam.

13	
14	
15	
16	
17	
18	
Subtotal	
1-12	
Total	

Letter Grade ________

1. Let $f(x) = (3x^2 + 5x - 6)^3$. Then $f'(1)$ is equal to

(A) 12
(B) $3(3 + 5 - 6)^2$
(C) $(3 + 5 - 6)^4$
(D) $3(3 + 5 - 6)^2(6 + 5)$
(E) 144

2. The tangent line to the curve $y = x^3 - 2x^2 + 2x + 1$ at the point (2,5) has equation

(A) $y - 5 = (3x^2 - 4x + 2)(x - 2)$
(B) $y = 5x/2$
(C) $y - 5 = (12 - 8 + 2)(x - 2)$
(D) $x - 2 = (12 - 8 + 2)(y - 5)$
(E) $y - 5 = -6(x - 2)$

3. $\lim_{x \to 2} \dfrac{x^2 - 4}{x^3 - 8}$ is equal to

(A) 0
(B) 1/2
(C) 1/3
(D) 2/5
(E) 2/3

4. Let $f(x)$ be defined by

$$f(x) = \begin{cases} |x-2|, & \text{if } x < 3 \\ (x-2)^2, & \text{if } 3 \leq x \leq 4 \\ x-4, & \text{if } x > 4. \end{cases}$$

Then f is continuous

(A) except at $x = 2$;

(B) except at $x = 3$;

(C) except at $x = 4$;

(D) except at $x = 3$ and $x = 4$;

(E) except at $x = 2,\ x = 3$ and $x = 4$.

5. Let $f(x) = 2x^3 - 3x^2 - 12x$. Then

$$f'(x) = 6(x-2)(x+1) \text{ and } f''(x) = 6(2x-1).$$

Then the absolute maximum of $f(x)$ on the interval $[-2, 2]$ occurs

(A) at $x = -2$

(B) at $x = -1$

(C) at $x = 0$

(D) at $x = 2$

(E) nowhere

6. The equation $7x^2y^3 - 5xy^2 - 4y = 7$ defines y implicitly as a function of x. Find dy/dx.

(A) $\dfrac{14xy^3 + 5y^2}{4 - 21x^2y^2 - 10xy}$

(B) $\dfrac{5y^2 - 14xy^3}{21x^2y^2 - 10xy - 4}$

(C) $\dfrac{5y^2 + 14xy^3}{21x^2y^2 - 10xy - 4}$

(D) $(7x^2y^3 - 5xy^2)/4$

(E) 0

7. Suppose that $f(x)$ is a function with first derivative $f'(x) = \dfrac{x^2 - 3x}{(x-1)^2}$. Then $f(x)$ is increasing on

(A) $(-\infty, 1)$ and $[3, \infty)$

(B) $[0, 1)$ and $[3, \infty)$

(C) $(-\infty, 0]$ and $(1, 3]$

(D) $(-\infty, 0]$ and $(1, \infty)$

(E) $(-\infty, 0]$ and $[3, \infty)$

8. Let $f(x) = (x+2)e^x$. Then, using the Mean Value Theorem, we can conclude that there is at least one number c between 1 and 4 such that $f'(c)$ is equal to

(A) $2e^4 - e$

(B) $3e^4 - (3/2)e$

(C) $3e^4 + (3/2)e$

(D) $6e^4 - 3e$

(E) $6e^4$

9. $\displaystyle\int \frac{x^{1/2}+x}{x^{5/2}}dx =$

(A) $-\dfrac{1}{x} - \dfrac{2}{\sqrt{x}} + C$

(B) $\dfrac{\frac{3}{2}x^{3/2} + \frac{1}{2}x^2}{\frac{7}{2}x^{7/2} + C}$

(C) $\dfrac{3}{x^3} + \dfrac{5}{2x^{5/2}} + C$

(D) $\dfrac{1}{x} + \dfrac{2}{\sqrt{x}} + C$

(E) $-\dfrac{1}{x} - \dfrac{1}{2\sqrt{x}} + C$

10. Let $f(x) = \int_2^x \sqrt{7t^2+8}dt$. Then $f'(2) =$

(A) 0

(B) 2

(C) 6

(D) $\frac{7}{3}$

(E) $\frac{1}{12}$

11. The substitution $x = u^2$ turns $\int_2^3 \tan \sqrt{x} dx$ into

(A) $\int_{\sqrt{2}}^{\sqrt{3}} \tan u \; du$

(B) $\int_{\sqrt{2}}^{\sqrt{3}} 2u \tan u \; du$

(C) $\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{2} u \tan u \; du$

(D) $\int_4^9 \tan u \; du$

(E) $\int_4^9 2u \; \tan u \; du$

12. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{2}$

(D) 2π

(E) $\frac{\pi}{6}$

Hand-graded part

13.(16 points) a) If $x^2 + y^2 = 2$, find $\frac{dy}{dx}$.

b) Find an equation of the tangent to the circle $x^2 + y^2 = 2$ at the point $(1, -1)$.

14.(17 points) Show that the equation $5x - 7 - \sin x = 0$ has exactly one real root.

15.(17 points) A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$. Find the distance traveled during the time period $1 \leq t \leq 4$.

16.(18 points) Find the area of the largest rectangle that can be inscribed in a semicircle of radius 1. Explain why your answer is an absolute maximum.

17.(18 points) Consider the function

$$f(x) = \frac{x^2}{x^2 - 4}.$$

We have

$$f'(x) = \frac{-8x}{(x^2-4)^2} \text{ and } f''(x) = \frac{8(4+3x^2)}{(x^2-4)^3}.$$

a) Find the domain of $f(x)$.

b) Determine the x-intercept and y-intercept of $y = f(x)$.

c) Determine the horizontal and vertical asymptotes of $y = f(x)$.

d) Determine the critical points, intervals of increase or decrease of $f(x)$.

e) Determine the concavity intervals of $f(x)$ and points of inflection.

f) Sketch the curve $y = f(x)$.