# Math 1271   Final Exam   Fall 2005

Last name: _____ First name: _____

Student ID: _____ Discussion section: _____ TA: _____

I certify that the answer on this exam are my own, produced in accordance with all University and Institute of Technology policies on Scholastic Conduct. Signature: _____

## Read and follow these instructions

This booklet contains 15 pages, including this cover page. Check to see if any are missing. PRINT all of the information requested above, and sign your name. Put your initials on the top of every page, in case the pages become separated. You are not allowed to have any sort of books, notes, cell phones, calculators, computers, or any other electronic devices outside of your backpack in the exam room.

There are 15 machine-graded problems worth 10 points each of 6 hand-graded problems worth 25 points each, together for a total of 300 points. You have 3 hour to do the problems.

Instructions for machine-graded part (Questions 1-15):
You MUST use a soft pencil (No. 1 or No 2) to answer this part. Do not fold or tear the answer sheet. Carefully enter all the requested information according to the instructions you receive. Do not make any stray marks on the answer sheet. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. **Notice regarding the machine graded portion of this exam:** Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. *Therefore work and answers must be clearly shown on the test booklet.*

Instructions for the hand-graded part (Questions 16-21): You must show all steps in your solutions and make your reasoning clear with English sentences to earn credit. Simplification of answers not required. SHOW ALL WORK.

After you finish both parts of the exam: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing DOWN. Have your ID card in your hand when turning in your exam.

Total _____

Letter Grade _____

| Problem | | |
|---------|---|---|
| 16 | | |
| 17 | | |
| 18 | | |
| 19 | | |
| 20 | | |
| 21 | | |
| Subtotal | | |
| 1-15 | | |

**1.** Find $y' = \frac{dy}{dx}$ where $x^2 + y^2 = e^{xy}$.

    (A) $y' = (ye^{xy} - 2x)/(2y - xe^{xy})$

    (B) $y' = (ye^{xy} - 2x)/(2y + xe^{xy})$

    (C) $y' = (e^{xy} - x)/2y$

    (D) $y' = e^{xy}/(2x - 2y)$

    (E) None of the above.

**2.** The derivative of $\int_{x^2}^{10} \frac{\sin(e^t)\, dt}{t^2+1}$ is:

    (A) $[\sin(e^{x^2})]/(e^{2x} + 1)$

    (B) $-[\sin(e^x)]/(e^{2x} + 1)$

    (C) $-[2x\sin(e^{x^2})]/(x^4 + 1)$

    (D) $10 + 2[x\sin(e^{x^2})]/(x^4 + 1)$

    (E) $[\sin(e^{10})]/101 + [\sin(e^{x^2})]/(x^4 + 1)$

**3.** Use the properties of the integral $I = \int_1^2 \sqrt{x^4 + 9\sin^2 x}\, dx$ to find the best upper and lower bounds on $I$:

(A) $1 \leq I \leq 4$
(B) $1 \leq I \leq 5$
(C) $0 \leq I \leq 4$
(D) $1 \leq I \leq 2$
(E) $-3 \leq I \leq 3$

**4.** Find $\lim_{t \to 0} \frac{\sin(\sin(\sin t))}{t}$:

(A) $+\infty$
(B) 0
(C) 1
(D) 2
(E) Does not exist.

**5.** Write as an integral and evaluate: $\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^7$.

   (A) 1

   (B) $\frac{1}{2}$

   (C) $\frac{1}{4}$

   (D) $\frac{1}{8}$

   (E) 4

**6.** You are given a function $f(x)$ that is continuous and differentiable on the closed interval $[-1, 2]$. You are also told that $f(-1) = -5$ and $f(2) = 7$. Which of the following statements about $f(x)$ is NOT true?

   (A) $f(x)$ has an absolute max for some $x$ on $[-1, 2]$.

   (B) There is a point $c$ with $-1 < c < 2$ where $f(c) = 0$.

   (C) There is a point $c$ with $-1 < c < 2$ where $f'(c) = 4$.

   (D) $f'(x) > 0$ for some values of $x$ between $-1$ and $2$.

   (E) Any of the statements $A$, $B$, $C$, $D$ can be false depending on what other properties $f(x)$ has.

**7.** The volume enclosed by a sphere of radius $r$ is $V = \frac{4}{3}\pi r^3$. Joe is blowing up a balloon to celebrate his completion of Math 1271. He is blowing air into it at the rate of 3 cubic inches per minute. How fast is the radius $r$ changing when the volume is $36\pi$?

(A) $\frac{\pi}{36}$ inches/minute

(B) $\frac{1}{12\pi \sqrt[3]{\pi}}$ inches/minute

(C) $108\pi$ inches/minute

(D) $36\pi$ inches/minute

(E) $\frac{1}{12\pi}$ inches/minute

**8.** Where is the function $f(x) = 2x^6 - 5x^4 + 20$ concave DOWN?

(A) For all values of $x$.

(B) $\{|x| > 1\}$

(C) $\{-1 < x < 0\}$ and $\{1 < x\}$

(D) $\{x < -1\}$ and $\{0 < x < 1\}$

(E) $\{|x| < 1\}$

**9.** Using linear approximation, estimate $\sqrt[3]{7.7}$.

    (A) 2.25

    (B) 2.025

    (C) 1.975

    (D) 1.75

    (E) 1.995

**10.** We are given the $45°$ right triangle whose shorter sides have length two. Find the area of the largest rectangle you can put inside of it.

    (A) $\frac{1}{2}$

    (B) 1

    (C) $\frac{3}{2}$

    (D) 2

    (E) 4

**11.** Find $\lim_{x \to 0+} x \ln x$

   (A) 0
   (B) $+\infty$
   (C) 1
   (D) $e$
   (E) Does not exist

**12.** The value of $\int_0^1 \frac{dx}{1+x^2}$ is

   (A) $\ln 2$
   (B) 1
   (C) $\pi$
   (D) $\pi/2$
   (E) $\pi/4$

**13.** Find $\lim_{x \to +\infty} \frac{x \sin x}{\ln x}$

(A) $+\infty$

(B) 0

(C) 1

(D) Does not exist

(E) $2\pi$

**14.** The value of $\int_0^{\frac{\pi}{4}} \tan x \, dx$ is

(A) $(\ln 2)/2$

(B) $\ln(1/\sqrt{2})$

(C) $\pi/4$

(D) $-\ln(\pi/4)$

(E) $\ln 2$

**15.** Find $\lim_{x \to 0} \frac{(\sin 2x)(\sin 4x)(\sin 6x)}{x^3}$

(A) $+\infty$

(B) $\frac{1}{48}$

(C) $1$

(D) $48$

(E) $0$

**HAND GRADED PART:** the next six problems count 25pts each. Simplification not required.

**16.** Using ONLY the definition of derivative by means of the difference quotient, find the derivative of $f(x) = x^3$.

**17.** Find the total area of the bounded regions between the $x$-axis and the graph of $y = x(x-1)(x-2)$.

**18.** Find the average value of the function $f(x) = (1 + \sqrt{x})^2$ on the interval $[0, 4]$.

**19.** Write down the equation of the tangent line to the graph of $f(x) = x^{x^2}$ at $x = 1$.

**20.** Joe was hired by a bagel company. His first assignment was to find the volume of flour needed to make each of the company's large bagels. The size of its bagels was to be determined by rotating the circle $(x-7)^2+y^2 = 9$ with center $(7,0)$ about the $y$-axis. First Joe used the disk method to write down the volume as an integral. He set up the integral by forming washers (rings) of thickness $\Delta y$ and areas of the form $\pi(g(y)^2 - h(y)^2)$ for the appropriate functions $g(y), h(y)$. At first he thought the integral he got was impossible to evaluate easily but then he remembered that $\int_0^3 \sqrt{9 - y^2}\, dy$ is related to the area of a circle. Find the volume of the bagel. You do not have to use Joe's method but it is easier than others.

**21.** Examine the function $f(x) = xe^{-2x^2}$. (i) Find where $f(x)$ is increasing and decreasing and has local max or min. (ii) Find where $f(x)$ is concave up and down and has points of inflection. (iii) Find $\lim f(x)$ as $x \to +\infty$ and $x \to -\infty$. (iv) Find where $f(x) > 0$ and $f(x) < 0$. (v) Finally, sketch the graph of $y = f(x)$.