

Math 1271 Final Exam Spring 2005

Last name: _____ First name: _____

ID: _____ Discussion section: _____ TA: _____

I certify that the answers on this exam are my own, produced in accordance with all University and Institute of Technology policies on Scholastic Conduct. Signature: _____

Read and follow these Instructions

This booklet contains 15 pages, including this cover page. Check to see if any pages are missing. **PRINT** all of the information requested above, and sign your name. Put your initials on the top of every page, in case the pages become separated.

There are 15 machine-graded problems worth 10 points each and 6 hand-graded problems worth 25 points each, together for a total of 300 points. You have 3 hours to do the problems.

Instructions for machine-graded part (Questions 1–15):

You **MUST** use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet. Carefully enter all of the requested information according to the instructions you receive. **DO NOT MAKE STRAY MARKS ON THE ANSWER SHEET.** When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle on the answer sheet. If you erase something, do so completely. Each question has exactly one correct answer. If you give two different answers, your response will be marked wrong.

Notice regarding the machine-graded portion of this exam: Either the student or the School of Mathematics may for any reason request a regrading of the machine-graded part. All regrades will be based upon responses in the test booklet, and not on the machine-graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations, will be marked wrong on the regrade. Therefore, *work and answers must be clearly shown in the test booklet.*

Instructions for hand-graded part (Questions 16–21):

You must show all steps in your solutions and make your reasoning clear with English sentences to earn credit. **SHOW ALL WORK.**

After you finish both parts of the exam:

Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked “GENERAL PURPOSE ANSWER SHEET” facing **DOWN**. Have your ID card in your hand when you turn in your exam.

16	
17	
18	
19	
20	
21	
Subtotal	
1–15	
Total	

Letter Grade: _____

1. Which of the following is NOT equal to $\int_0^2 \sqrt{4-x^2} dx$?

A. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \sqrt{4 - (2i)^2/n^2}$

B. π

C. $F(2) - F(0)$ where $F'(x) = \sqrt{4-x^2}$

D. The area of a half circle of radius 2

E. $\lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \frac{2}{n} \sqrt{4 - (2i)^2/n^2}$

2. Suppose that the function f is continuous and differentiable on the closed interval $[-1, 2]$. Assume that $f(-1) = -5$ and $f(2) = 7$. Which of the following is NOT true of the function f ?

A. f achieves an absolute maximum on the interval $[-1, 2]$.

B. There is a point c in the interval $[-1, 2]$ where $f(c) = 0$.

C. There is a point c in the interval $(-1, 2)$ where $f'(c) = 4$.

D. The above statements must all be true for f .

E. The above statements can all be false for f .

3. Which of the following describes the set of real numbers on which the function $f(x) = \ln(1 + x^2)$ is concave down?

- A. $0 < x < 1$
- B. $|x| > 1$
- C. $-1 < x < 1$
- D. $x < -1$
- E. $x > 1$

4. Evaluate $\int \frac{\sin x}{(\cos x)^{\frac{1}{3}}} dx$.

- A. $\frac{2}{3}(\cos x)^{-\frac{2}{3}} + C$
- B. $\frac{1}{3}(\cos x)^{\frac{3}{2}} + C$
- C. $-\frac{3}{2}(\cos x)^{\frac{2}{3}} + C$
- D. $-\frac{1}{3}(\cos x)^{\frac{1}{3}} + C$
- E. $-\frac{3}{2}(\sin x)^{-\frac{2}{3}} + C$

5. The difference of two numbers is 20. What is the smallest possible value for the product of these two numbers?

- A. -100
- B. -240
- C. -400
- D. 100
- E. 240

6. Evaluate $\lim_{x \rightarrow -8} \frac{\sqrt{1-x} - 3}{2 + x^{\frac{1}{3}}}$.

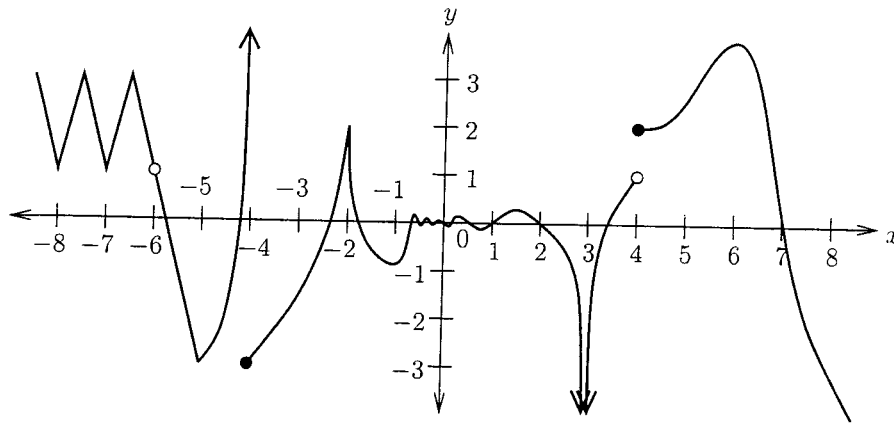
- A. -2
- B. ∞
- C. $-\infty$
- D. -4
- E. 8

7. The area of the region lying between the curves $y = 2x - x^2$ and $y = -x$ is equal to which of the following?

- A. 2
- B. 9
- C. 4
- D. 27
- E. $9/2$

8. Suppose a particle traveling along a straight line is $\frac{1}{1+t}$ meters from the origin after t seconds. The average velocity of the particle away from the origin between times $t = 0$ and $t = 3$ is:

- A. $-\frac{1}{4}$ meters per second
- B. $\frac{5}{12}$ meters per second
- C. $\frac{1}{3} \ln(4)$ meters per second
- D. $-\frac{4}{9}$ meters per second
- E. $\frac{1}{4}$ meters per second



Problems 9 and 10 refer to the graph $y = f(x)$ depicted above. An open circle means that the function's value is not the height of that circle. A solid circle means that the function's value equals the height of that circle.

9. The number of values a in the interval $[-8, 8]$ for which $\lim_{x \rightarrow a} f(x)$ does not exist is:
- A. 1
 - B. 2
 - C. 3
 - D. 4
 - E. 5 or more
10. If f' is the derivative of f , then $f'(x) < 0$ when:
- A. $x = -6$
 - B. $1 < x < 2$
 - C. $x = 4$
 - D. $x > 6$
 - E. none of the above

11. If $f(x) = e^{3x} - e^{-3x}$, then what is the 1271th derivative $f^{(1271)}(x)$?

A. $3^{1271}f(x)$

B. $e^{3813x} - e^{-3813x}$

C. $3^{1271}f(-x)$

D. $1271f^{(1270)}(x)$

E. $3^{1271}(e^{3x} + e^{-3x})$

12. The derivative of the function $f(t) = \ln(\ln(\ln t))$ is:

A. $\frac{1}{t}$

B. $\frac{1}{\ln t}$

C. $\frac{1}{t \ln(t) \ln(\ln t)}$

D. $\frac{1}{\ln(\ln(\ln t))}$

E. $\frac{1}{t \ln(t)^3}$

13. If $f(x) = \int_2^{x+1} t\sqrt{7+t^2}dt$ then what is $f'(2)$?

A. 0

B. $\frac{1}{3}(64 - 11\sqrt{11})$

C. 1

D. $2\sqrt{11}$

E. 12

14. If $f(x) = \frac{\sqrt{x} + 1}{\sqrt{x}}$ then $|f(x) - 1| < \frac{1}{10}$ for all $x > N$ if

A. $N = -10$.

B. $N = 10$.

C. $N = -100$.

D. $N = 100$.

E. $N = \frac{1}{10}$.

15. If $e^{x/y} = x + y$, then what is $\frac{dy}{dx}$?

A. $\frac{xy}{x^2 + xy + y^2}$

B. $\frac{xe^{x/y} + y^2}{ye^{x/y} - y^2}$

C. $\frac{xy}{xe^{x/y} - y^2}$

D. $\frac{y}{x} - \frac{2}{xy^2e^{x/y}}$

E. $\frac{ye^{x/y} + y^2}{xe^{x/y} - y^2}$

16. A television camera is positioned 4 kilometers from the base of a rocket launching pad. Suppose that the camera is always aimed at a rocket, and that the rocket moves straight upward. If θ is the angle of elevation of the camera, then $\theta = 0$ when the rocket is on the launch pad, and θ increases as the rocket rises.

(i) Express $\tan(\theta)$ in terms of the height of the rocket.

(ii) What is $\cos(\theta)$ when the rocket is 3 kilometers off the ground?

(iii) If the rocket is rising at 500 meters per second when it is 3 kilometers high, then how fast is θ increasing (in radians per second) at that moment? Express your answer as a fraction in lowest terms.

17. Estimate $\int_{-1}^1 (5x^2 + 1)dx$ using a Riemann sum with 5 subintervals of equal length. Use the left endpoints of these subintervals for your sample points. Express your answer as a fraction in lowest terms.

18. To make a doughnut for breakfast, rotate about the y -axis the disk bounded by $(x - 7)^2 + y^2 = 9$, centered at $(7, 0)$. Write a definite integral that gives the volume of your breakfast. Evaluate the integral (but don't eat the doughnut).

19. Let $f(x) = x(x^2 - 9x + 15)$. Find the absolute maximum and absolute minimum values attained by f on the interval $[0, 7]$. Determine the x -values where each of these extrema is attained.

20. Estimate $\sqrt[3]{66.4}$ by the method of linear approximation, using the fact that $4^3 = 64$. Be clear about which function you are linearly approximating and what its linear approximation is. At the end of your calculations, express your final answer as a single number in decimal notation.

21. Write an equation for the tangent line to the graph of $y = x^{\cos(\pi x)}$ when $x = 3$.