

Math 1271
Fall, 2003
FINAL EXAM

Name (Print) _____

Signature _____

T.A. Instructor _____ Discussion Section _____ I.D.# _____

READ AND FOLLOW THESE INSTRUCTIONS

This booklet contains 15 pages, including this cover page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. Books, notes and calculators are **NOT PERMISSIBLE**. Do your work in the blank spaces and back of pages of this booklet. Show all your work.

There are 15 machine-graded problems worth 10 points each and 6 hand-graded problems worth 25 points each, together for a total of 300 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-15):

You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. **DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET.** When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 16-21):

SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam. Either the student or the School of Mathematics may for any reason request a regrading of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. *Therefore work and answers must be clearly shown on the test booklet.*

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing **DOWN**. Have your ID card in your hand when turning in your exam.

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|----------|--|
| 16 | |
| 17 | |
| 18 | |
| 19 | |
| 20 | |
| 21 | |
| Subtotal | |
| 1-15 | |
| Total | |

Letter Grade _____

1. Let $f(x) = \frac{x^2 + 1}{x + 7}$. Then $f'(1)$ is equal to

(A) $\frac{7}{8}$

(B) $\frac{15}{8}$

(C) $\frac{15}{64}$

(D) $\frac{7}{64}$

(E) $\frac{7}{32}$

2. The tangent line to the curve $y^2 = 3x^2 + x + 2$ at the point $(-1, 2)$ has equation

(A) $y - 2 = (3x^2 + x + 2)(x + 1)$

(B) $y - 2 = -\frac{5}{4}(x + 1)$

(C) $y - 2 = -\frac{5}{4}(x - 1)$

(D) $y + 1 = 2(x - 2)$

(E) $y + 1 = -2(x - 2)$

3. The equation $\cos x + \sin y = \sin x \cos y$ defines y implicitly as a function of x . Then dy/dx is equal to

- (A) $\frac{\cos x \cos y + \sin x}{\cos y + \sin x \sin y}$
- (B) $\frac{\cos x \cos y - \sin x}{\cos y - \sin x \sin y}$
- (C) $\frac{\cos x \cos y + \sin x}{\cos y - \sin x \sin y}$
- (D) $\frac{\cos x \sin x - \cos y}{\sin y - \cos x \cos y}$
- (E) $\frac{\cos x + \cos y + \sin x}{\cos y - \sin x - \sin y}$

4. Let

$$f(x) = \begin{cases} x^3 - x, & \text{if } x < -1 \\ \frac{x^2 - 1}{x^2 + 3x + 2}, & \text{if } x \geq -1 \end{cases}$$

Then $\lim_{x \rightarrow (-1)^+} f(x)$ is equal to

- (A) Does not exist
- (B) 0
- (C) -2
- (D) -1
- (E) 1

5. Let $f(x)$ be defined by

$$f(x) = \begin{cases} -(x + 3), & \text{if } x < 1; \\ |x - 5|, & \text{if } 1 \leq x \leq 7; \\ (x + 1)^{1/3}, & \text{if } 7 < x. \end{cases}$$

Then f is continuous

- (A) except at $x = 1$;
- (B) except at $x = 7$;
- (C) except at $x = 1$ and $x = 7$;
- (D) everywhere;
- (E) only at $x = 7$.

6. Let $f(x) = 2x^3 - 6x + 1$. Then, on the interval $[-3, 2]$, what are the absolute minimum and absolute maximum values of $f(x)$?

- (A) -3 and 5
- (B) -1 and 1
- (C) -3 and 2
- (D) 3
- (E) -35 and 5

7. $\int_{4/5}^0 \frac{x}{\sqrt{1-x^2}} dx =$ (Hint: let $u = 1 - x^2$)

(A) $\sin^{-1}(4/5)$

(B) $1/5$

(C) $2/5$

(D) $-3/5$

(E) $4/5$

8. Let $f(x) = \int_x^7 (3t^2 - \sin t)^5 dt$. Then $f'(-2) =$

(A) $(12 + \sin(-2))^5$

(B) $-(12 - \sin(-2))^5$

(C) $+(12 - \sin(-2))^5$

(D) $(6(-2) - \cos(-2))^5$

(E) $5(12 - \sin(-2))^4(6(-2) - \cos(-2))$

9. Let $f(x) = \int_2^{x^2} t^t dt$. Then $f'(2) =$

- (A) 256
- (B) 512
- (C) 1024
- (D) $508(\ln(2) + 1)$
- (E) 0

10. The volume of the solid obtained by rotating about the x axis the region between the x axis and the curve $y = x^{4/5}$ from 0 to 1 equals:

- (A) π
- (B) $\frac{5\pi}{9}$
- (C) $\frac{4\pi}{5}$
- (D) $\frac{5\pi}{13}$
- (E) $\frac{8\pi}{5}$

11. Let R be the plane region bounded by the line $y = 6x$ and the curve $y = 3x^2$. Then the volume of the solid obtained by revolving R about the x -axis is given by

(A) $\pi \int_0^2 (36x^2 - 9x^4) dx$

(B) $\pi \int_0^2 (6x - 3x^2)^2 dx$

(C) $\pi \int_0^2 (y - 2y)^2 dy$

(D) $\pi \int_0^2 (3x^2 + 6x)^2 dx$

(E) $\int_0^2 (y^2 - 4y^2) dy$

12. $\int_1^3 x^{3/4}(x + x^2) dx$ equals

(A) $(\frac{4}{7}3^{7/4} - \frac{4}{7}) + (\frac{3^2}{2} - \frac{1}{2}) + (\frac{3^3}{3} - \frac{1}{3})$

(B) $(\frac{7}{4}3^{7/4} - \frac{7}{4}) + 2(3^2 - 1) + 3(3^3 - 1)$

(C) $(\frac{7}{4}3^{7/4} - \frac{7}{4}) + (\frac{3^3}{2} + \frac{3^3}{3}) - (\frac{1}{2} + \frac{1}{3})$

(D) $(\frac{11}{4}3^{11/4} + \frac{15}{4}3^{5/4}) - (\frac{11}{4} + \frac{15}{4})$

(E) $(\frac{4}{11}3^{11/4} + \frac{4}{15}3^{15/4}) - (\frac{4}{11} + \frac{4}{15})$

13. $\lim_{x \rightarrow 1} \frac{\sin(x^2-1)}{x-1}$ equals

- (A) 1
- (B) 2
- (C) π
- (D) 0
- (E) 0/0

14. $\frac{d}{dx} \tan(\sqrt{1+x^2})$ equals

- (A) $\frac{x}{\sqrt{1+x^2}} \tan(\sqrt{1+x^2})$
- (B) $2x \tan(\sqrt{1+x^2})$
- (C) $\frac{1}{2} \frac{x^2}{\sqrt{1+x^2}} \sec^2(\sqrt{1+x^2})$
- (D) $\frac{x}{\sqrt{1+x^2}} \sec^2(\sqrt{1+x^2})$
- (E) $\frac{x}{(1+x^2)^{3/2}}$

15. $\frac{d}{dx}(3^{x \ln x})$ equals

(A) $(3^{x \ln x})(1 + \frac{1}{x})$

(B) $(\ln 3)(3^{x \ln x})(1 + \frac{1}{x})$

(C) $(3^{x \ln x})(\ln x + 1)$

(D) $(3^{x \ln x})(\ln x)$

(E) $(\ln 3)(3^{x \ln x})(\ln x + 1)$

Hand-graded part

16. A solid is formed by revolving the circular disk $(x - 5)^2 + y^2 = 4$ about the y -axis. Set up but do not evaluate a definite integral for the volume of the solid.

17. Find the following limit or derivatives:

a)(12 points) Find the number b such that the average value of $f(x) = 1 - 6x + 3x^2$ on the interval $[0, b]$ is 3.

b)(13 points) Find all numbers c in the interval $[1,4]$ that satisfy the mean value theorem (for derivatives) for the function $f(x) = \frac{x}{x+2}$.

18. Find the following definite and indefinite integrals.

a)(10 pts) $\int_1^e \frac{\sqrt{\ln x}}{x} dx$

b)(15 pts) $\int \frac{x^2}{\sqrt{1-x}} dx$

19. Find the dimension of the rectangle of largest area that has its base on the x -axis and its other two vertices above the x axis and lying on the parabola $y = 8 - x^2$.

20. Consider the function

$$f(x) = xe^{x+1}$$

a. Give the domain of $f(x)$:

b. Determine the x-intercept and the y-intercept of $y = f(x)$;

c. Determine the horizontal and vertical asymptotes of $y = f(x)$ if any (L'Hospitals rule may be of use).

d. Determine the critical points and the intervals of increase or decrease of $f(x)$;

(Recall: Critical points are points where the derivative does not exist or the derivative exists and is zero.)

e. Determine the intervals of up and down concavity of $f(x)$ and points of inflection.

f. Sketch the curve $y = f(x)$.

21. Find the area between the x -axis and $y = |x^2 - x|$ in the interval $[-2, 5]$.