Name: _____

Signature: _____

Section: _____

Math 1271. Lecture 030 Practice Midterm Exam I

There are a total of 100 points on this exam. To get full credit for a problem you must show the details of your work. Answers unsupported by by an argument will get little credit. No books, notes, calculators, cell phones or other elecronic devices are allowed. Do all of your calculations on this test paper.

Problem	Score
1.	
2.	
3.	
4.	
5.	
Total:	

Problem 1 *Compute the limits. It is important that you show your work. The answer alone is not sufficient.*

a. (4 points)

$$\lim_{x \to 1} \sqrt{|x| - x}$$

b. (4 points)

$$\lim_{x \to -6} \frac{x+6}{x^2 - 36}$$

c. (4 points)

$$\lim_{x \to \pi} \frac{\sin x}{x}$$

d. (4 points)

$$\lim_{x \to -\infty} \frac{4x}{1 - x^2}$$

e. (4 points)

$$\lim_{x \to +\infty} \frac{x \cos x}{x + \cos x}$$

Problem 2 *Compute the derivatives $f'(x)$. It is not necessary to simplify your answer.*

a. (6 points)

$$f(x) = 5x^{10} - 24x^2 + 2x - 1$$

b. (7 points)

$$f(x) = \frac{2x+3}{\sqrt{x}-5}$$

c. (6 points)

$$f(x) = (3x^2 + 2x - 5)(6x^4 - 7x^2 + x - 2)$$

Problem 3 (20 points)
Find the points (x, y) *on the curve* $y = f(x) = x^3 - x^2 - x + 1$ *where the tangent to the curve is horizontal.*

Problem 4 (20 points)
Use the Intermediate Value Theorem for continuous functions to show that the polynomial $P(x) = x^5 - 10x$ *must have at least 3 distinct real roots, i.e., at least 3 distinct values of* x *such that* $P(x) = 0$. *You must justify your answer, but it is not necessary to actually find the roots.*

Problem 5 *Let*

$$y = f(x) = \frac{(x-2)^2(x+1)}{x-3}$$

with natural domain.

a. (4 points) *Find the vertical asymptotes of f.*

b. (4 points) *Find the horizontal asymptotes of f.*

c. (2 points) *Find the x-intercepts of f, i.e., the values of x such that $f(x) = 0$.*

d. (2 points) *Find the y-intercept of f.*

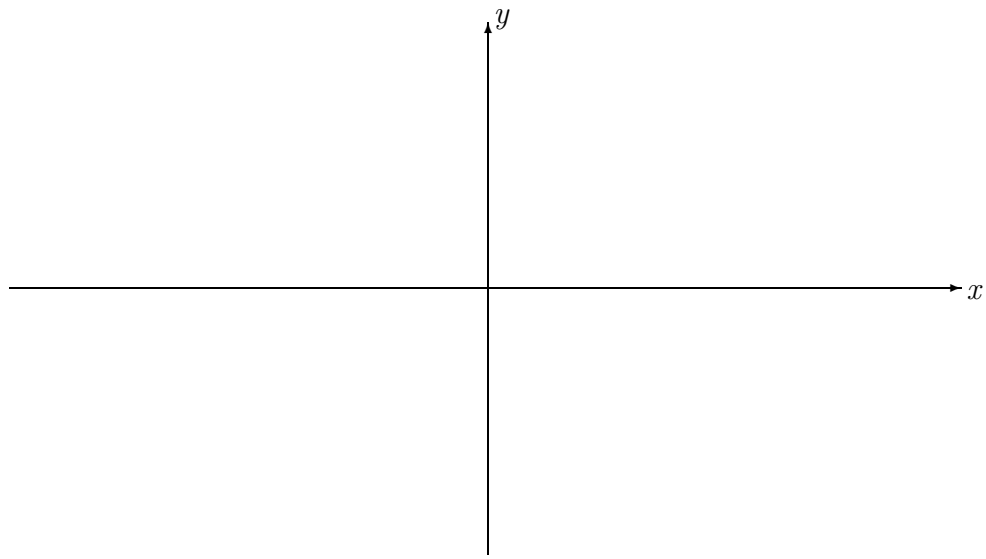

Figure 1: Graph of the curve $y = (x-2)^2(x+1)/(x-3)$

e. (8 points) *Sketch the curve, pointing out significant features such as the asymptotes, intercepts and behavior approaching infinity.*

Very Brief Answers

1a. 0

1b. $-1/12$

1c. 0

1d. 0

1e. Limit does not exist.

2a.

$$f'(x) = 50x^9 - 48x + 2$$

2b.

$$f'(x) = \frac{2(\sqrt{x} - 5) - (2x + 3)(\frac{1}{2\sqrt{x}})}{(\sqrt{x} - 5)^2}$$

2c.

$$f'(x) = (6x + 2)(6x^4 - 7x^2 + x - 2) + (3x^2 + 2x - 5)(24x^3 - 14x + 1)$$

3. Points $(-1/3, 32/27)$ and $(1, 0)$

4a. $P(0) = 0$ so obviously $x_0 = 0$ is a root. $P(1) = -9 < 0$ and $P(2) = 10 > 0$. Since P is continuous, the IVT says that P must have a root x_1, such that $1 < x_1 < 2$. $P(-1) = 9 > 0$ and $P(-2) = -12 < 0$. Since P is continuous, the IVT says that P must have a root x_2, such that $-2 < x_2 < -1$. Thus P must have at least 3 roots.

5a $x = 3$

5b. $\lim_{x\to+\infty} f(x) = +\infty$, $\lim_{x\to-\infty} f(x) = +\infty$. No horizontal asymptotes

5c. $x = -1, 2$

5d. $y = -4/3$

5e. f is positive for $x > 3$, $x < -1$. f is negative for $2 < x < 3$ and $-1 < x < 2$. $y \to +\infty$ as $x \to \pm\infty$