

# 6651 Review Final I

1. Convert the following integral to an integral with variable  $\theta$  using the substitution  $3x = 5 \sin \theta$ .

$$\int_{5/6}^{5\sqrt{2}/6} \frac{x^2 dx}{\sqrt{25 - 9x^2}}$$

2. Find an approximate value of the integral  $\int_2^5 \sqrt{8 + x^2} dx$  using the trapezoid rule with  $n = 6$ .

3. Suppose  $f(x)$  is a function such that  $|f''(x)| \leq 12$  for  $2 \leq x \leq 8$ . Find a bound for  $|E_T|$  for the following integral when  $n = 20$ . Recall that  $|E_T| \leq \frac{K(b-a)^3}{12n^2}$ .

$$\int_2^8 f(x) dx$$

- b) Given that  $|f''(x)| \leq 20$  for  $0 \leq x \leq 25$ . Find the smallest value of  $n$  such that the error  $E_T$  made when evaluating the following integral using the trapezoid rule satisfies  $|E_T| < 10^{-3}$ .

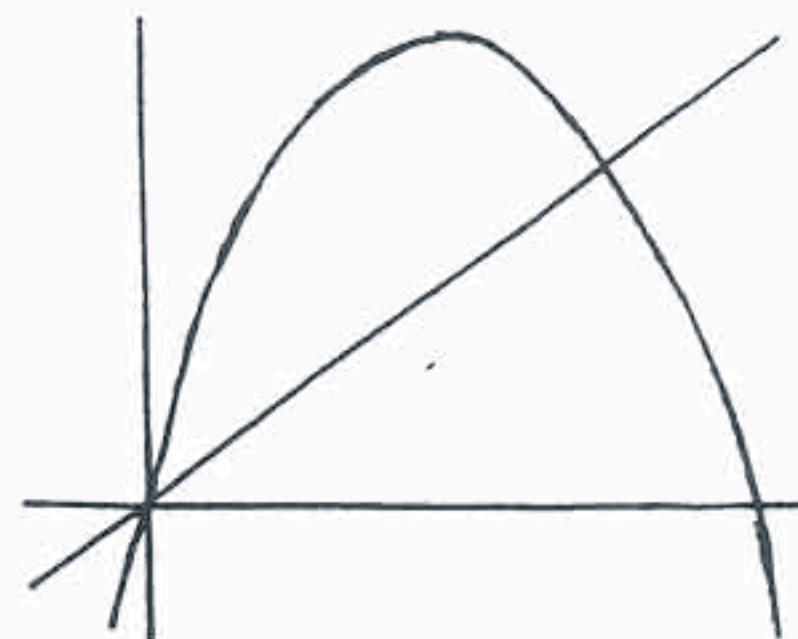
$$\int_3^{15} f(x) dx$$

4. If the following integral is convergent then evaluate it. If it is divergent, then explain why it is divergent.

$$\int_1^\infty \frac{x dx}{(x^2 + 8)^2}$$

## 6652 Review Final II

1. The region bounded by the parabola  $y = 6x - x^2$  and the line  $y = x$  is covered by a lamina of density  $\rho$ . Find  $M_x$ ,  $M_y$ , and the mass for this lamina.



2. Solve the initial value problem

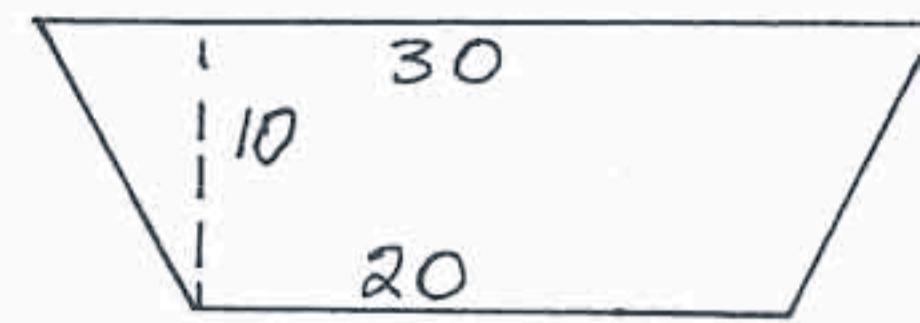
$$\frac{dy}{dx} = \frac{4}{x}y + x^6 \quad y(1) = \frac{7}{3}.$$

3. Solve the differential equation  $\frac{dy}{dx} = \frac{(2x+3)(y^2+9)}{y(x^2+3x)}$ . Solve for  $y^2$  in the solution.

4. A very large tank contains 400 gallons of water with 50 lbs of salt dissolved in the water. Brine containing  $3/2$  lbs of salt per gallon is pumped into the tank at the rate of 8 gallons per minute. The mixture is pumped out of the tank at the slower rate of 6 gallons per minute. Find an expression for the amount of salt in the tank at time  $t$ .

### 6653 Review Final III

1. The end of a pool is in the shape of a trapezoid with equal sides. The trapezoid is 30 ft long at the top and 20 feet long at the bottom. The pool is 10 feet deep. If the pool is full of water, find the total force on the end of the pool.



2. Find the area of the region which is outside the circle  $r = 6 \cos \theta$  but inside the cardioid  $r = 2 + 2 \cos \theta$ .
3. Find the area of the region which is inside the cardioid  $r = \sin \theta - 1$  and below the  $x$  axis.
4. Evaluate the following indefinite integral. Show all steps.

$$\int \frac{x^2 + 13x + 4}{(x^2 + 4)(x + 3)} dx$$

## 6654 Review Final IV

1. Show that the following alternating series is convergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{8n+5}{n(2n+1)}$$

2. First, find values of  $p$  and  $c$  such that  $\frac{8n+7}{n(n^2+1)} \leq \frac{c}{n^p}$ . Show that the series is convergent using the comparison theorem.

$$\sum_{n=1}^{\infty} \frac{8n+7}{n(n^2+1)}$$

3. For what values of  $x$  is the following power series convergent? divergent?

$$\sum_{n=0}^{\infty} \frac{2^n(n+2)x^n}{5^n(3n+5)}$$

4. The Taylor polynomial  $T_5(x)$  for  $(4+x)^{3/2}$  is

$$(4+x)^{3/2} \approx 8 + 3x + \frac{3}{16}x^2 - \frac{1}{128}x^3 + \frac{3}{4096}x^4 - \frac{3}{32768}x^5$$

Use this polynomial to find Taylor polynomials for  $(4+x)^{1/2}$  and  $(4+x)^{5/2}$ .

## 6655 Review Final V

1. Find the vector function  $\vec{R}(t)$  whose graph is the tangent line to the curve which is the graph of the vector function  $\vec{r}(t) = (3t^2 + 5)\vec{i} + (2t^2 - 3t)\vec{j}$  at the point  $(17, 2)$ .

2. Find the equation of the plane through the lines  $\text{Line A: } \begin{cases} x = 2t \\ y = 4t \\ z = t \end{cases}$  and  $\text{Line B: } \begin{cases} x = 1 + s \\ y = 1 - s \\ z = 2s \end{cases}$

2. If  $N = 8$  for what values of  $x$  is  $\left| \arctan x - \sum_{k=0}^N (-1)^k \frac{x^{2k+1}}{2k+1} \right| < 10^{-6}$ .

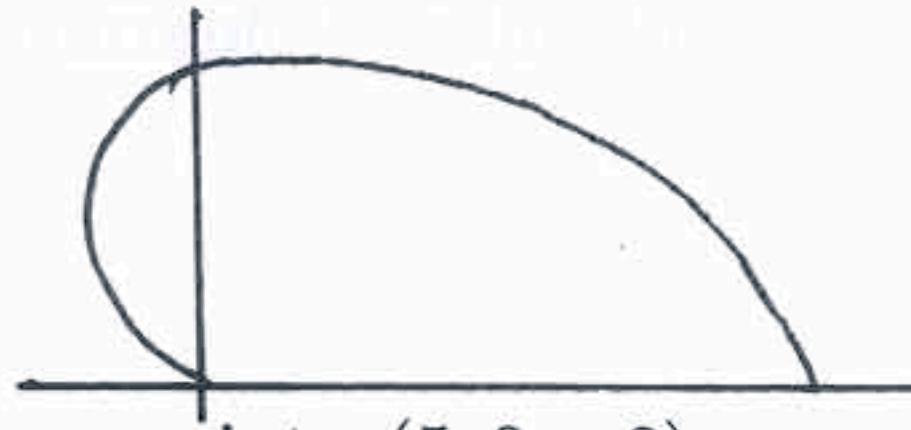
3. If  $|x| \leq 0.8$  find the smallest value of  $N$  such that

$$\left| \sin x - \sum_{k=0}^N (-1)^k \frac{x^{2k+1}}{(2k+1)!} \right| < 10^{-8}.$$

4. Given  $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k}$  and  $\vec{b} = 4\vec{i} - 3\vec{k}$  find the angle between  $\vec{a}$  and  $\vec{b}$ . Find  $\text{Proj}_{\vec{a}} \vec{b}$ .

## 6656 Review Final VI

1. A section of the graph of the cardioid  $r = \cos \theta - 1$  is shown. What values of  $\theta$  correspond to this section of the graph?



2. Find the equation of the plane through the three points  $(5, 0, -3)$ ,  $(2, -4, 7)$  and  $(1, 4, -6)$ .

3. Consider the following geometric series. What number term is  $59049/16$ ?

$$64 + 96 + 144 + 216 + \dots + (59049/16)$$

4. Find the point where the two lines that are graph of the following vector functions intersect.

$$\begin{aligned}\vec{r}(t) &= (4t - 2)\vec{i} + (-2t)\vec{j} + (6t - 15)\vec{k} \\ \vec{R}(s) &= (3s + 14)\vec{i} + (5s + 5)\vec{j} + (-2s - 4)\vec{k}\end{aligned}$$

5. The following are the rectangular coordinates of some points. Find four sets of polar coordinates for these points.

(a)  $(-4\sqrt{3}, 4)$       (b)  $(-6, -6)$

6. Find the sum  $-13 - 7 - 1 + 5 + 11 \dots + 1199$ .

# 6651 Review Final I

1. Convert the following integral to an integral with variable  $\theta$  using the substitution  $3x = 5 \sin \theta$ .

$$\frac{25}{27} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^2 \theta d\theta$$

$$\int_{5/6}^{5\sqrt{2}/6} \frac{x^2 dx}{\sqrt{25 - 9x^2}}$$

$dx = \frac{5}{3} \cos \theta d\theta$   
 $3 \left( \frac{5\sqrt{2}}{6} \right) = 5 \sin \theta$   
 $\theta = \frac{\pi}{4}$   
 $3 \left( \frac{5}{6} \right) = 5 \sin \theta$   
 $\theta = \frac{\pi}{6}$

2. Find an approximate value of the integral  $\int_2^5 \sqrt{8 + x^2} dx$  using the trapezoid rule with  $n = 6$ .

$$f(2) = 3,4641$$

$$f(2.5) = 3,77492$$

$$f(3) = 4,12311$$

$$f(3.5) = 4,5$$

$$f(4) = 4,89898$$

$$f(4.5) = 5,31507$$

$$13,6082$$

$$\Delta x = \frac{5-2}{6} = \frac{1}{2}$$

$$f(5) = 5,74456$$

3. Suppose  $f(x)$  is a function such that  $|f''(x)| \leq 12$  for  $2 \leq x \leq 8$ . Find a bound for  $|E_T|$  for the following integral when  $n = 20$ . Recall that

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}.$$

$$\int_2^8 f(x) dx \quad \frac{|2(8-2)|^3}{12(20)^2} = \frac{27}{50}$$

- b) Given that  $|f''(x)| \leq 20$  for  $0 \leq x \leq 25$ . Find the smallest value of  $n$  such that the error  $E_T$  made when evaluating the following integral using the trapezoid rule satisfies  $|E_T| < 10^{-3}$ .

$$\frac{20(12)^3}{12n^2} < 10^{-3}$$

$$\int_3^{15} f(x) dx$$

$$n^2 > (2880)10^3$$

$$n > 1697,06$$

$$n = 1698$$

4. If the following integral is convergent then evaluate it. If it is divergent, then explain why it is divergent.

$$\int_1^b \frac{x dx}{(x^2+8)^2} = -\frac{1}{2} (x^2+8)^{-1} \Big|_1^b = \frac{1}{18} - \frac{1}{2(b^2+8)}$$

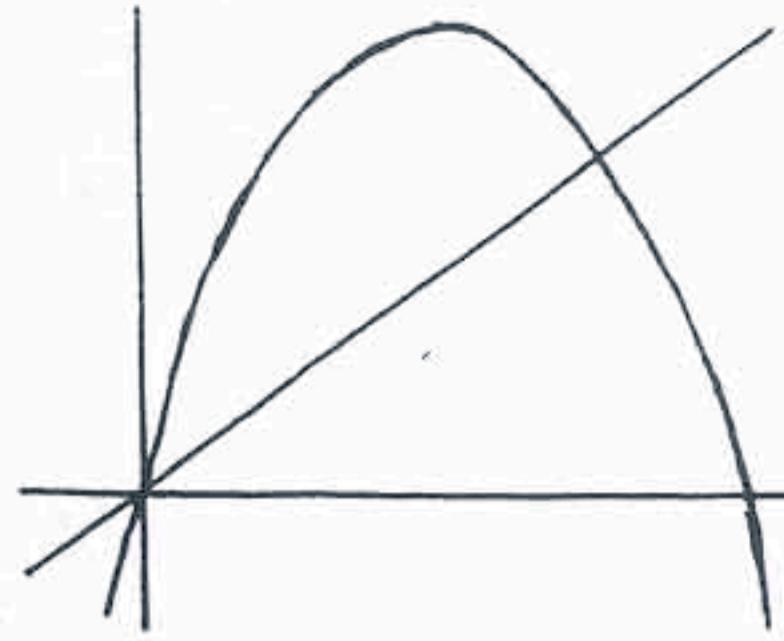
$$\lim_{n \rightarrow \infty} \left[ \frac{1}{18} - \frac{1}{2(b^2+8)} \right] = \frac{1}{18}$$

Integral Converges

## 6652 Review Final II

1. The region bounded by the parabola  $y = 6x - x^2$  and the line  $y = x$  is covered by a lamina of density  $\rho$ . Find  $M_x$ ,  $M_y$ , and the mass for this lamina.

See Attached sheet



2. Solve the initial value problem

$$\frac{dy}{dx} - \frac{4}{x}y = x^6 \quad \frac{dy}{dx} = \frac{4}{x}y + x^6 \quad y(1) = \frac{7}{3}.$$

$$x^{-4} \frac{dy}{dx} - 4x^{-5}y = x^2 \\ x^{-4}y = \frac{x^3}{3} + C \\ y = \frac{x^7}{3} + Cx^4$$

3. Solve the differential equation  $\frac{dy}{dx} = \frac{(2x+3)(y^2+9)}{y(x^2+3x)}$ . Solve for  $y^2$  in the solution.

$$\frac{y dy}{y^2 + 9} = \frac{2x+3}{x^2+3x} dx \quad y^2 = -9 + C(x^2+3x)^2$$

$$\frac{1}{2} \ln(y^2+9) = \ln|x^2+3x| + \ln C$$

$$y^2 + 9 = C(x^2+3x)^2 \quad x \neq 0, x \neq -3$$

4. A very large tank contains 400 gallons of water with 50 lbs of salt dissolved in the water. Brine containing  $3/2$  lbs of salt per gallon is pumped into the tank at the rate of 8 gallons per minute. The mixture is pumped out of the tank at the slower rate of 6 gallons per minute. Find an expression for the amount of salt in the tank at time  $t$ .

See Attached sheet

$$1. \text{ Mass} = \rho \int_0^5 (6x - x^2 - x) dx = \frac{5}{6}\rho = \frac{125}{6}\rho$$

$$M_x = \frac{\rho}{2} \int_0^5 [(6x - x^2)^2 - x^2] dx$$

$$= \frac{\rho}{2} \left[ \frac{35x^3}{3} - 3x^4 + \frac{x^5}{5} \right]_0^5 = \frac{5^4}{6}\rho = \frac{625}{6}\rho$$

$$M_y = \rho \int_0^5 x[(6x - x^2) - x] dx$$

$$\rho \left[ \frac{5x^3}{3} - \frac{x^4}{4} \right]_0^5 = \rho 5^4 / 12 = \frac{625}{12}\rho$$

$$4. \text{ Rate in} = \left( \frac{3}{2} \text{ lb/gal} \right) (8 \text{ gal/min}) = 12 \text{ lbs/min}$$

$$\text{Rate out} = (6 \text{ gal/min}) \left( \frac{y}{400+t} \text{ lbs/gal} \right) = \frac{3y}{200+t}$$

$$\frac{dy}{dt} = 12 - \frac{3}{200+t} y$$

$$(200+t)^3 y = 3(200+t)^4 + C$$

$$y = 3(200+t) + C(200+t)^{-3}$$

$$C = -550(200)^3$$

$$y = 3(200+t) - 550(200)^3(200+t)^{-3}$$

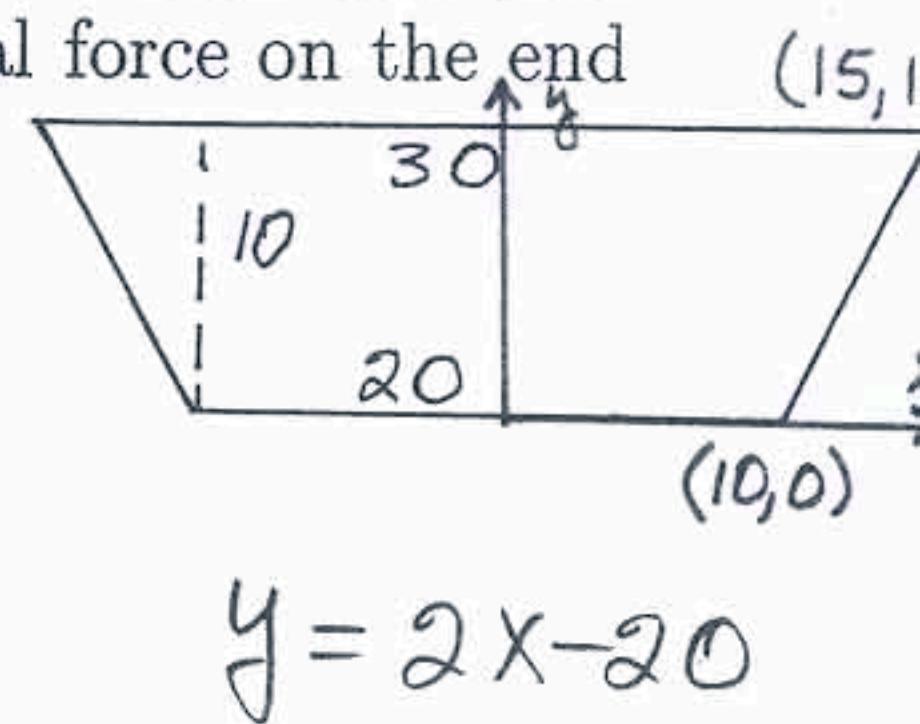
## 6653 Review Final III

1. The end of a pool is in the shape of a trapezoid with equal sides. The trapezoid is 30 ft long at the top and 20 feet long at the bottom. The pool is 10 feet deep. If the pool is full of water, find the total force on the end of the pool.

$$\Delta F = (10-y)(2x)\Delta y$$

$$\text{Force} = \int_0^{10} \rho(10-y)(y+20) dy$$

$$= \frac{7(10)^3}{6} \rho = \frac{3500}{3} \rho$$



2. Find the area of the region which is outside the circle  $r = 6 \cos \theta$  but inside the cardioid  $r = 2 + 2 \cos \theta$ .

See Attached Sheet

3. Find the area of the region which is inside the cardioid  $r = \sin \theta - 1$  and below the  $x$  axis.

See Attached sheet

4. Evaluate the following indefinite integral. Show all steps.

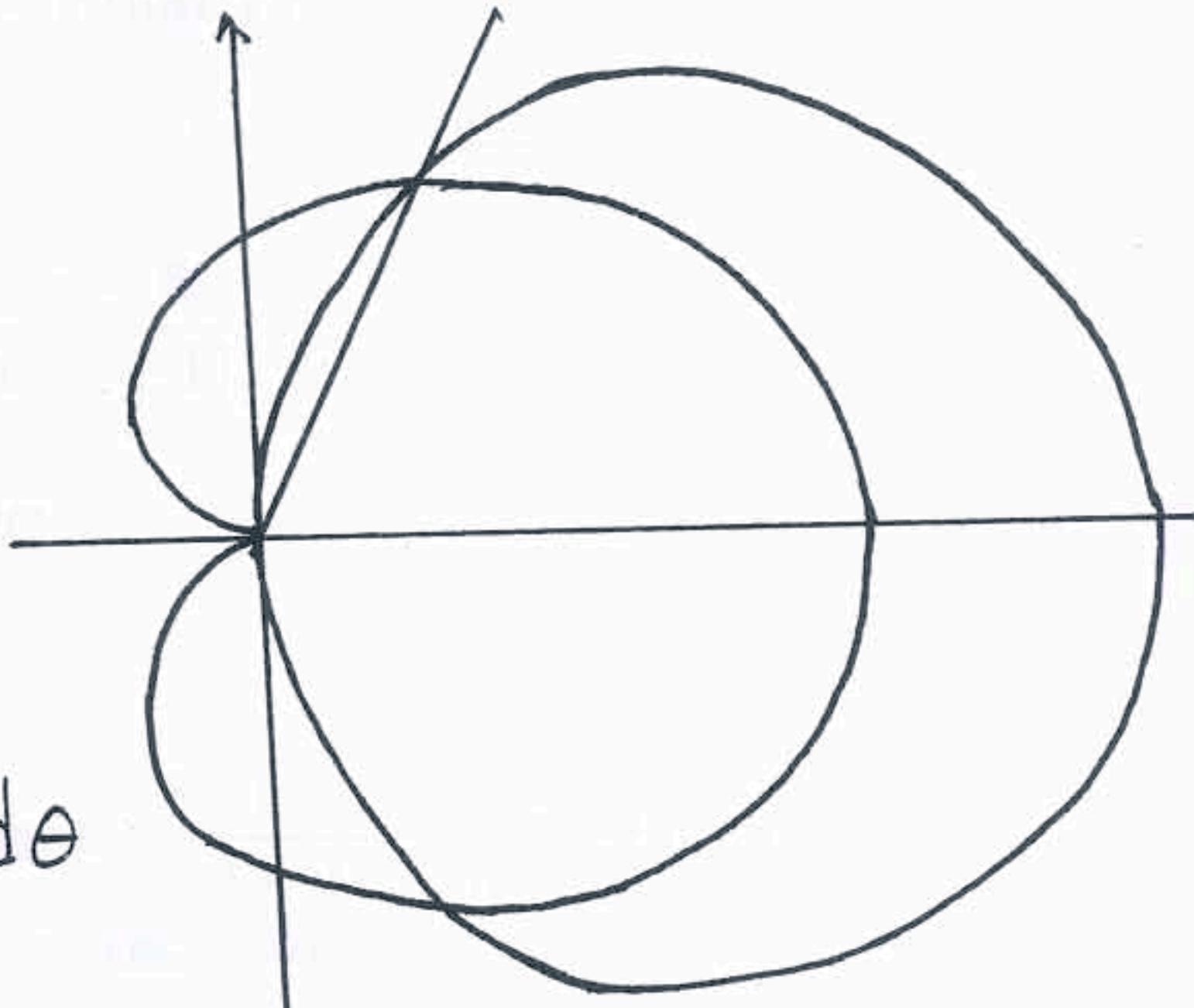
$$\int \frac{x^2 + 13x + 4}{(x^2 + 4)(x + 3)} dx$$

$$\int \left[ \frac{3x+4}{x^2+4} - \frac{2}{x+3} \right] dx$$

$$= \frac{3}{2} \ln(x^2+4) + 2 \arctan \frac{x}{2} - 2 \ln|x+3| + C$$

$$2. \int_{\frac{\pi}{3}}^{\pi} (2+2\cos\theta)^2 d\theta$$

$$- \int_{\frac{\pi}{3}}^{\pi} (6\cos\theta)^2 d\theta$$

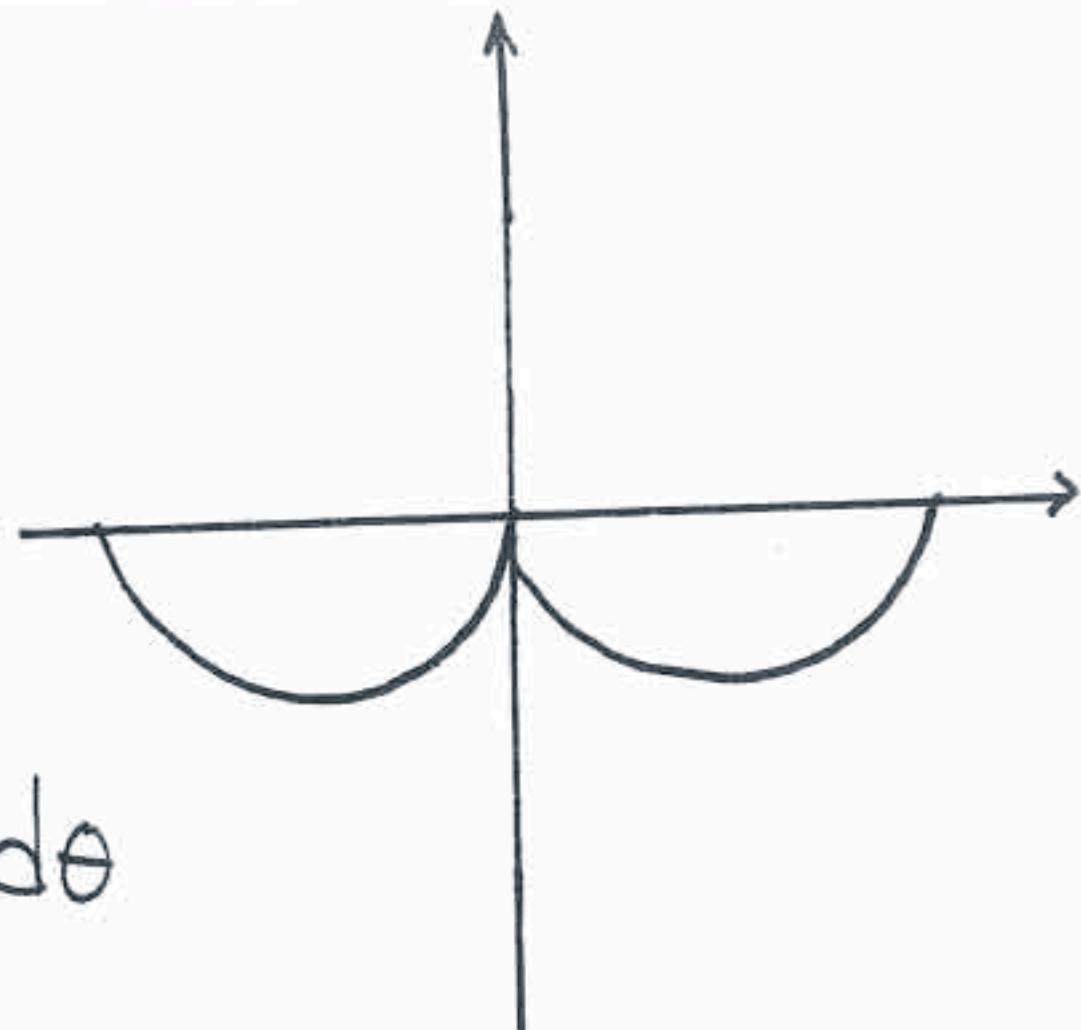


$$= \int_{\frac{\pi}{3}}^{\pi} (6 + 8\cos\theta + 2\cos 2\theta) d\theta$$

$$- \int_{\frac{\pi}{3}}^{\pi} (18 + 18\cos 2\theta) d\theta$$

$$= \pi$$

$$3. \frac{1}{2} \int_0^{\pi} (\sin\theta - 1)^2 d\theta$$



$$= \frac{1}{2} \int_0^{\pi} \left( \frac{3}{2} - \frac{1}{2}\cos 2\theta - 2\sin\theta \right) d\theta$$

$$= \frac{3\theta}{4} - \frac{1}{8}\sin 2\theta + \cos\theta \Big|_0^{\pi}$$

$$= \frac{3\pi}{4} - 2$$

## 6654 Review Final IV

1. Show that the following alternating series is convergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{8n+5}{n(2n+1)}$$

See Attached Sheet

2. First, find values of  $p$  and  $c$  such that  $\frac{8n+7}{n(n^2+1)} \leq \frac{c}{n^p}$ . Show that the series is convergent using the comparison theorem.

$$\sum_{n=1}^{\infty} \frac{8n+7}{n(n^2+1)}$$

See Attached Sheet

3. For what values of  $x$  is the following power series convergent? divergent?

$$\sum_{n=0}^{\infty} \frac{2^n(n+2)x^n}{5^n(3n+5)}$$

See Attached Sheet

4. The Taylor polynomial  $T_5(x)$  for  $(4+x)^{3/2}$  is

$$(4+x)^{3/2} \approx 8 + 3x + \frac{3}{16}x^2 - \frac{1}{128}x^3 + \frac{3}{4096}x^4 - \frac{3}{32768}x^5$$

Use this polynomial to find Taylor polynomials for  $(4+x)^{1/2}$  and  $(4+x)^{5/2}$ .

See Attached Sheet

1. The Alternating Series theorem says:

If  $\lim_{n \rightarrow \infty} \frac{8n+5}{n(2n+1)} = 0$  AND  $\frac{8n+13}{(n+1)(2n+3)} < \frac{8n+5}{n(2n+1)}$ ,

then  $\sum_{n=1}^{\infty} (-1)^n \frac{8n+5}{n(2n+1)}$  converges. Clearly,  $\lim_{n \rightarrow \infty} \frac{8+5/n}{2n+1} = 0$ .

$n(2n+1)(8n+13) < (8n+5)(n+1)(2n+3)$  is the same as  
 $16n^3 + 34n^2 + 13n < 16n^3 + 50n^2 + 49n + 15$  which is  
true if  $0 < 16n^2 + 36n + 15$  which is clearly true.

We conclude from those three statements taken  
together that  $\sum_{n=1}^{\infty} (-1)^n \frac{8n+5}{n(2n+1)}$  converges.

2.  $\frac{8n+7}{n(n^2+1)} = \frac{8+7/n}{n^2+1} \approx \frac{8}{n^2}$ . Now  $\frac{8n+7}{n(n^2+1)} < \frac{9}{n^2}$

is true if  $8n^2 + 7n < 9n^2 + 9$  which is true if  
 $7n < n^2 + 9$  which is true for  $n=1$  and  $n \geq 6$ .

The series  $\sum_{n=1}^{\infty} \frac{9}{n^2}$  converges since it is a  
P-Series with  $P=2$  and  $C=9$ . The comparison

test theorem says: If  $\sum_{n=1}^{\infty} \frac{9}{n^2}$  converges AND

if  $\frac{8n+7}{n(n^2+1)} < \frac{9}{n^2}$ , then  $\sum_{n=1}^{\infty} \frac{8n+7}{n(n^2+1)}$  converges.

We conclude that  $\sum_{n=1}^{\infty} \frac{8n+7}{n(n^2+1)}$  converges.

3. the Ratio is

$$\frac{a_{n+1}}{a_n} = \frac{2^{n+1}(n+3)x^{n+1}}{5^{n+1}(3n+8)} \cdot \frac{5^n(3n+5)}{2^n(n+2)x^n}$$
$$= \frac{2(n+3)(3n+5)}{5(3n+8)(n+2)} x$$

$$L = \lim_{n \rightarrow \infty} \frac{2(1+\frac{3}{n})(3+\frac{5}{n})|x|}{5(3+\frac{8}{n})(1+\frac{1}{n})} = \frac{6|x|}{15} = \frac{2|x|}{5}$$

the Ratio test theorem says:

If  $|x| < \frac{5}{2}$ , then  $\sum_{n=0}^{\infty} \frac{2^n(n+2)x^n}{5^n(3n+5)}$  converges.

If  $|x| > \frac{5}{2}$ , then  $\sum_{n=0}^{\infty} \frac{2^n(n+2)x^n}{5^n(3n+5)}$  diverges.

4. Differentiating both sides

$$\frac{3}{2}(4+x)^{\frac{1}{2}} \approx 3 + \frac{3}{16}(2x) - \frac{1}{128}(3x^2) + \frac{3}{4096}(4x^3)$$
$$- \frac{3}{32768}$$

$$(4+x)^{\frac{1}{2}} \approx 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \frac{1}{512}x^3 - \frac{5}{16384}x^4$$

4. cont.

Integrating both sides

$$\frac{(4+x)^{5/2}}{5/2} \sim C + 8x + 3\frac{x^2}{2} + \frac{3}{16}\frac{x^3}{3} - \frac{1}{128}\frac{x^4}{4} + \frac{3}{4096}\frac{x^5}{5} - \frac{3}{32768}\frac{x^6}{6}$$

$$(4+x)^{5/2} \sim \frac{5}{2}C + 20x + \frac{15}{4}x^2 + \frac{5}{32}x^3 - \frac{5}{1024}x^4 + \frac{3}{8192}x^5 - \frac{5}{131072}x^6$$

$$x=0 \text{ gives } \frac{5}{2}C = 32$$

$$(4+x)^{5/2} = 32 + 20x + \frac{15}{4}x^2 + \frac{5}{32}x^3 - \frac{5}{1024}x^4 + \frac{3}{8192}x^5 - \frac{5}{131072}x^6$$

# 6655 Review Final V

1. Find the vector function  $\vec{R}(t)$  whose graph is the tangent line to the curve which is the graph of the vector function  $\vec{r}(t) = (3t^2 + 5)\vec{i} + (2t^2 - 3t)\vec{j}$  at the point  $(17, 2)$ .  $\vec{r}(2) = 17\vec{i} + 2\vec{j}$   $t=2$

$$\vec{r}'(t) = (6t)\vec{i} + (4t-3)\vec{j} \quad \vec{r}'(2) = 12\vec{i} + 5\vec{j}$$

$$\vec{R}(t) = (12t + 17)\vec{i} + (5t + 2)\vec{j}$$

2. If  $N = 8$  for what values of  $x$  is  $\left| \arctan x - \sum_{k=0}^N (-1)^k \frac{x^{2k+1}}{2k+1} \right| < 10^{-6}$ .

$$\left| \sum_{k=N+1}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} \right| < \frac{|x|^{2N+3}}{2N+3} \quad \text{for } |x| < 1$$

$$\text{we Need } \frac{|x|^{19}}{19} < 10^{-6} \quad |x| < 0.5643$$

3. If  $|x| \leq 0.8$  find the smallest value of  $N$  such that

$$\left| \sin x - \sum_{k=0}^N (-1)^k \frac{x^{2k+1}}{(2k+1)!} \right| < 10^{-8}$$

We Must find  $N$  such that

$$\frac{(0.8)^{2N+3}}{(2N+3)!} < 10^{-8}$$

$$N = 4$$

$$\frac{(0.8)^9}{9!} = 3.7 \times 10^{-7} > 10^{-8}$$

$$\frac{(0.8)^{11}}{11!} = 2.2 \times 10^{-9} < 10^{-8}$$

4. Given  $\vec{a} = 3i - 5j + 2k$  and  $\vec{b} = 4i - 3k$  find the angle between  $\vec{a}$  and  $\vec{b}$ . Find  $\text{Proj}_{\vec{a}} \vec{b}$ .

$$\cos \theta = \frac{6}{5\sqrt{30}}$$

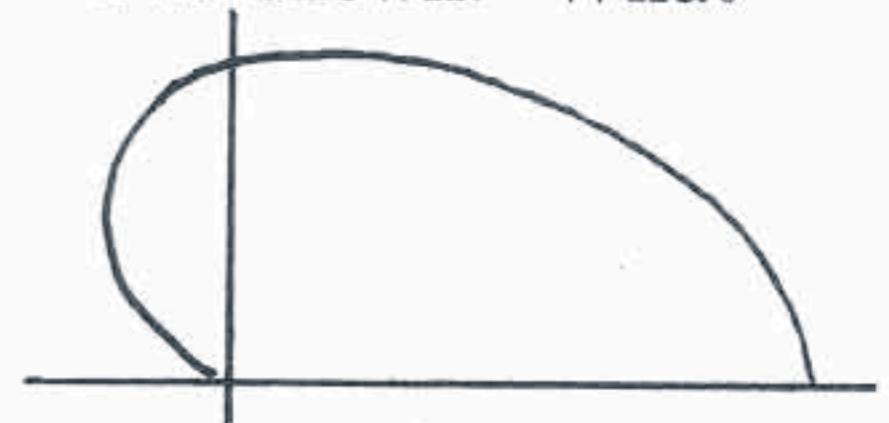
$$\theta = 1.375$$

$$\text{Proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \vec{a}$$

$$= \frac{6}{38} (3i - 5j + 2k)$$

# 6656 Review Final VI

1. A section of the graph of the cardioid  $r = \cos \theta - 1$  is shown. What values of  $\theta$  correspond to this section of the graph?



$$\pi \leq \theta \leq 2\pi$$

2. Find the equation of the plane through the three points  $(5, 0, -3)$ ,  $(2, -4, 7)$  and  $(1, 4, -6)$ .

See Attached Sheet

3. Consider the following geometric series. What number term is  $59049/16$ ?

$$a_1 = 64 \quad 64 + 96 + 144 + 216 + \dots + (59049/16)$$

$$n = \frac{3}{2} \quad 64 \left(\frac{3}{2}\right)^{n-1} = \frac{59049}{16} \quad n-1=10$$

$$\left(\frac{3}{2}\right)^{n-1} = \frac{59049}{1024} = \left(\frac{3}{2}\right)^{10} \quad n=11$$

4. Find the point where the two lines that are graph of the following vector functions intersect.

$$\vec{r}(t) = (4t - 2)\vec{i} + (-2t)\vec{j} + (6t - 15)\vec{k}$$

$$\vec{R}(s) = (3s + 14)\vec{i} + (5s + 5)\vec{j} + (-2s - 4)\vec{k}$$

See Attached Sheet

5. The following are the rectangular coordinates of some points. Find four sets of polar coordinates for these points.

$(a) (-4\sqrt{3}, 4)$	$(b) (-6, -6)$	$(b) (6\sqrt{2}, 5\sqrt{4})$	$(-6\sqrt{2}, \frac{\pi}{4})$
$(a) (8, 5\pi/6)$	$(-8, 11\pi/6)$	$(6\sqrt{2}, -\frac{3\pi}{4})$	$(-6\sqrt{2}, -\frac{7\pi}{4})$
$(8, -7\pi/6)$	$(-8, -\pi/6)$		

6. Find the sum  $-13 - 7 - 1 + 5 + 11 \dots + 1199$ .

$$a_1 = -13 \quad d = 6$$

$$S_{203} = \frac{203}{2} [-13 + 1199]$$

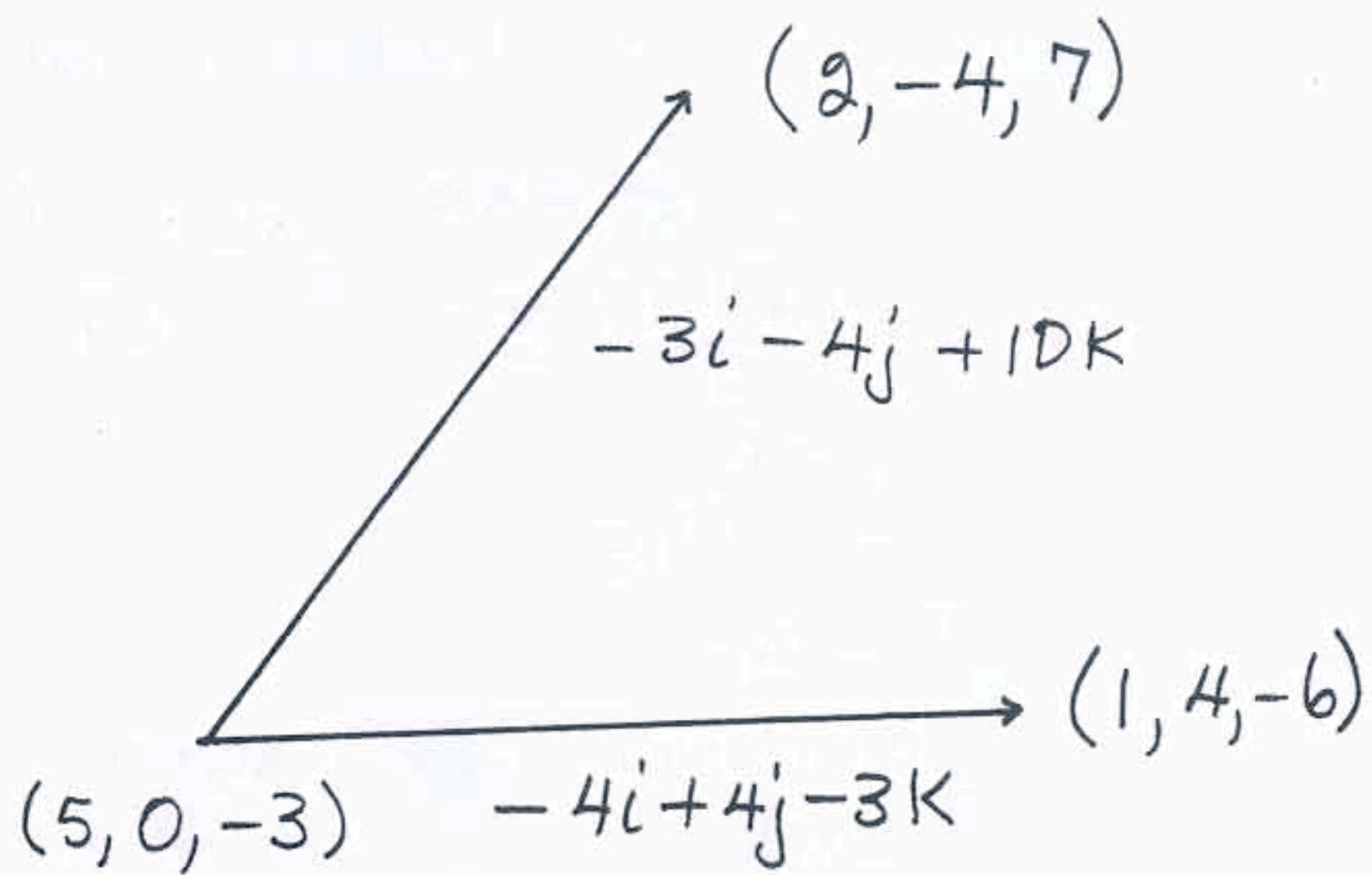
$$199 = -13 + (n-1)6$$

$$= 120, 379$$

$$n = 203$$

2.

$$\begin{vmatrix} i & j & k \\ -3 & -4 & 10 \\ -4 & 4 & -3 \end{vmatrix}$$



$$= -28i - 49j - 28k$$

$$-28(x-5) - 49(y-0) - 28(z+3) = 0$$

$$28x + 49y + 28z = 56$$

4. Must find  $s$  and  $t$  such that

$$4t - 2 = 3s + 14$$

$$s = -2$$

$$-2t = 5s + 5$$

$$t = \frac{5}{2}$$

$$6t - 15 = -2s - 4$$

Intersection Point  $(8, -5, 0)$ .

# Review Final #1

①  $3x = 5 \sin \theta \Rightarrow 3dx = 5 \cos(\theta) d\theta \Rightarrow dx = \frac{5}{3} \cos(\theta) d\theta$

$$x = \frac{5}{3} \sin(\theta) = \frac{5\sqrt{2}}{6} \quad \text{for } \sin(\theta) = \frac{\sqrt{2}}{2} \quad \text{and } \theta = \frac{\pi}{4}$$

$$x = \frac{5}{3} \sin(\theta) = \frac{5}{6} \quad \text{for } \sin(\theta) = \frac{1}{2} \quad \text{and } \theta = \frac{\pi}{6}$$

2<sup>nd</sup>: Then:  $x^2 dx = \left(\frac{5}{3} \sin \theta\right)^2 \frac{5}{3} \cos \theta d\theta = \left(\frac{5}{3}\right)^3 \sin^2(\theta) \cos(\theta) d\theta$

$$\text{and } \sqrt{25 - x^2} = \sqrt{25 - 25 \sin^2 \theta} = 5\sqrt{1 - \sin^2(\theta)} = 5 \cos \theta.$$

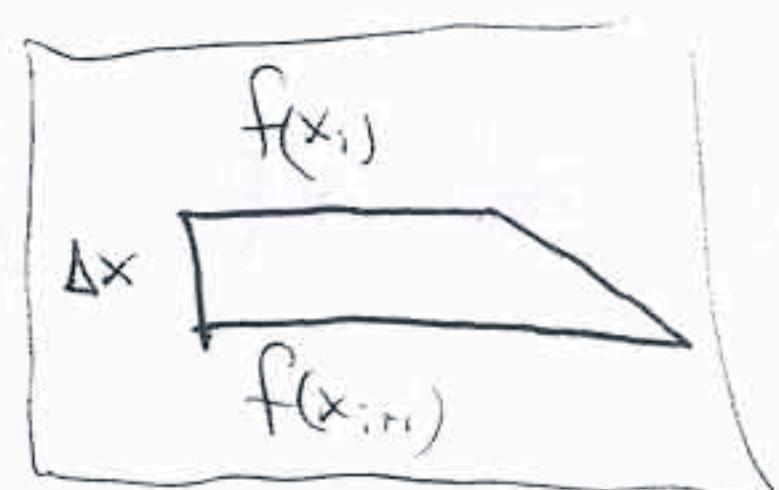
So: the integral becomes

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\left(\frac{5}{3}\right)^3 \sin^2(\theta) \cos(\theta) d\theta}{5 \cos(\theta)} = \frac{5^2}{3^3} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^2(\theta) d\theta$$

② What is the trapezoid rule?

$$\int_a^b f(x) dx \approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

Since the area of a trapezoid



$$\frac{\Delta x (f(x_i) + f(x_{i+1}))}{2}$$

... in Stewart page 518.

$\Delta x = \frac{b-a}{n}$ , in this case  $\frac{5-2}{6} = \frac{1}{2}$  and we have

$$\int_2^5 \sqrt{8+x^2} dx \approx \frac{1}{4} [f(2) + 2f(2.5) + 2f(3) + 2f(3.5) + 2f(4) + 2f(4.5) + f(5)]$$

and you have the numerical value of 13.6082.

Note: the points in the interval are  $x_0 + i\Delta x$  for  $0 \leq i \leq n$ .

③ This is Simple plug and chug.

They give you the formula & you use it.

a)  $k=12, b=8, a=2$ , and  $n=20$  so

$$|E| \leq \frac{12(8)^3}{12 \cdot 20^2} = \frac{6^3}{20^2} = \frac{27}{50}$$

b) Here you have to think maybe a little bit.

Just set up  $\frac{20(25)^3}{12 \cdot n^2} < 10^{-3}$  & solve

For  $n$  remembering to round up to the nearest integer.

⑨ In all such problems you should give the following pattern for our answer. Integral exists if the limit does.

$$\int_1^{\infty} \frac{x dx}{(x^2+8)^2} = \lim_{t \rightarrow \infty} \int_1^t \frac{x dx}{(x^2+8)^2}. \quad \text{Then, Since}$$

$$\int \frac{x dx}{(x^2+8)^2} = -\frac{1}{2} \frac{1}{x^2+8}, \quad \text{we have}$$

$$\int_1^{\infty} \frac{x dx}{(x^2+8)^2} = -\frac{1}{2} \left| \lim_{t \rightarrow \infty} \frac{1}{x^2+8} \right| = -\frac{1}{2} \left( \lim_{t \rightarrow \infty} \frac{1}{t^2+8} - \frac{1}{9} \right)$$

$$= \frac{1}{18}.$$

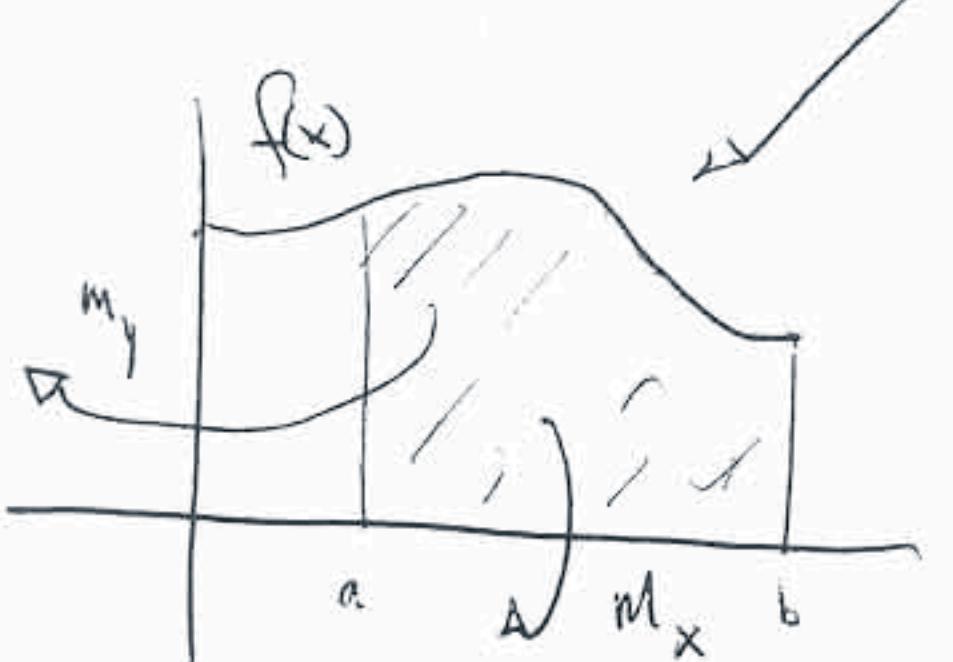
So: the integral is convergent & this is the value.

END OF REVIEW #1

## Review Final #2

① What are the formulas for moment? You must memorize them.

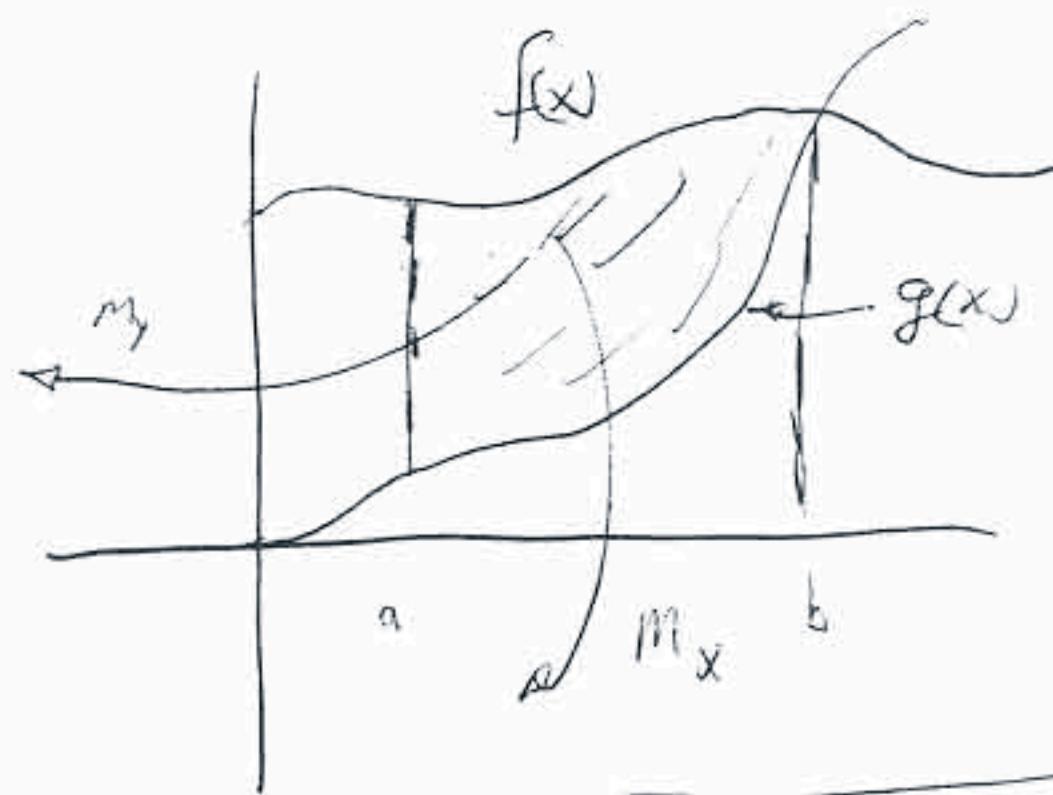
$$M_y = \rho \int_a^b x f(x) dx \quad M_x = \frac{\rho}{2} \int_a^b (f(x))^2 dx$$



For 2 functions, i.e. the region between  
you do the following change..

$$M_y = \rho \int_a^b x (f(x) - g(x)) dx$$

$$M_x = \frac{\rho}{2} \int_a^b [f(x)]^2 - [g(x)]^2 dx$$



MAKE SURE you DON'T DO This  $\rightarrow M_x = \frac{\rho}{2} \int_a^b (f(x) - g(x))^2 dx$   
Very Very BAD

So: For this problem we just want  $a \neq b$  and then  
we are basically done..

The functions intersect when  $x = 6x - x^2$ ; i.e.  $0 = 5x - x^2 = x(5-x)$

So  $a=0$  &  $b=5$ .

$$M_y = \rho \int_0^5 x(6x-x^2-x) dx = \rho \int_0^5 x(5x-x^2) dx = \frac{525}{12} \rho$$

$$M_x = \frac{\rho}{2} \int_0^5 ((6x-x^2)^2 - x^2) dx = \frac{625}{6} \rho$$

$$\text{and Mass} = \rho \int_0^5 (6x-x^2-x) dx = \frac{125}{6} \rho.$$

Very easy if you remember the formulas.

② This is a linear 1<sup>st</sup> order diff eq & requires the method of the integrating factor.

Remember: For  $\frac{dy}{dx} + p(x)y = Q(x)$ ,  $\int p(x) dx$

$$y = \frac{1}{I(x)} \int Q(x) I(x) dx \quad \text{where } I(x) = e^{\int p(x) dx}$$

So:  $\frac{dy}{dx} - \frac{4}{x}y = x^6$ ;  $p(x) = -\frac{4}{x}$   $Q(x) = x^6$

$$\int -\frac{4}{x} dx = -4 \ln(x)$$

$$I(x) = e^{-4 \ln(x)} = e^{\ln(x^{-4})} = \frac{1}{x^4} \quad \text{and so we get}$$

$$y = x^4 \int x^6 dx = x^4 \int x^2 dx = x^4 \left( \frac{x^3}{3} + C \right) = \frac{x^7}{3} + Cx^4$$

If you forgot to add the constant here, you will probably have major points deducted on the final.

Finally:  $y(1) = \frac{7}{3}$ , so  $\frac{7}{3} = \frac{1}{3} + C \Rightarrow C = 2$

So the final solution is  $y = \frac{x^7}{3} + 2x^4$ .

③ This is a Separable equation  $\nexists$  requires a simpler different method.

In general:  $\frac{dy}{dx} = g(y) f(x) \Rightarrow \frac{dy}{g(y)} = f(x) dx \nexists$  you integrate

$\frac{dy}{dx} = \frac{(2x+3)(y^2+9)}{y(x^2+3x)}$  is Separable, so we separate

$\frac{y dy}{y^2+9} = \frac{(2x+3) dx}{x^2+3x}$  and then integrate ..

$\int \frac{y dy}{y^2+9} = \frac{1}{2} \ln|y^2+9| - \int \frac{(2x+3) dx}{x^2+3x} = \ln|x^2+3x| + C_A$

most add constant here

To Simplify matters we move the  $\frac{1}{2}$  over to get

$$\ln|y^2 + a| = 2 \ln|x^2 + 3x| + 2C = \ln|(x^2 + 3x)^2| + 2C$$

$$y^2 + a = K(x^2 + 3x)^2$$

④

Basic Situation is the following:

$S(t)$  is the Salt,  $W(t)$  is the water,

$\frac{ds}{dt} = a - b \frac{s(t)}{w(t)}$  where you are basically given

$a$  &  $b$ , i.e. the rates in  $\frac{\text{#}}{\text{time}}$  out.

note:  $b$  is the rate out of the water, not the salt.

You will probably have to solve for the water function  
Since it will probably not be constant, although it may  
be.

$$q = \text{rate in} = \left( \frac{3}{2} \frac{\text{lbs}}{\text{gal}} \right) \left( 8 \frac{\text{gal}}{\text{min}} \right) = \left( 12 \frac{\text{lbs}}{\text{min}} \right)$$

$$b = \text{rate out of Water} = 6$$

Py 4

$$\text{So: } \frac{ds}{dt} = 12 - 6 \frac{s(t)}{w(t)}.$$

Now: all you need is the water function  $\beta$ . You can solve a 1<sup>st</sup> order linear system (i.e. use the integrating factor).

$$w'(t) = 8 - 6 = 2, \text{ so } w(t) = \int 2 dt = 2t + C \text{ where } C = w(0) = 400.$$

$$\text{So: } \frac{ds}{dt} = 12 - 6 \frac{s(t)}{2t+400} = 12 - \frac{3s(t)}{t+200} \text{ and we get}$$

$$\frac{ds}{dt} + \frac{3}{t+200}s(t) = 12, \quad \overline{I}(t) = e^{\int \frac{3 dt}{t+200}} \text{ and}$$

$$S(t) = \frac{1}{\overline{I}(t)} \int \overline{I}(t) \cdot 12 dt = \frac{12}{\overline{I}(t)} \int \overline{I}(t) dt.$$

$$\overline{I}(t) = (t+200)^3 \text{ and } S(t) = \frac{3}{(t+200)^3} \left( (t+200)^4 + C \right)$$

$$= 3(t+200) + \frac{12 \cdot C}{(t+200)^3}.$$

Now: You want to find  $B \cdot C$ ; and you use  
the initial conditions -

$$S(0) = 50 = B(200) + \frac{B \cdot C}{200^3} \quad \text{and}$$

$$3C = -550(200)^3 \quad \dagger$$

$$S(t) = B(t+200) - \frac{550(200)^3}{(t+200)^3},$$

END of Review 2

## Review Final 15

① YOU MUST SET THESE PROBLEMS UP WISELY & CAREFULLY.

Always use the symmetry (if there is any) and if you aren't expert in using the similar triangle stuff, just find the linear point slope.

So: you need depth and length

depth is  $10-y$  and length is  $2x$ , but you need  $x$  in terms of  $y$  so you need to know the ~~line~~....

Slope is  $\frac{10-0}{15-10} = \frac{10}{5} = 2$ . Point Slope form

$$y-0=2(x-10) \text{ and } x = \frac{y}{2} + 10 = \frac{y+20}{2}$$

Then: you integrate  $P \int_0^{10} \text{depth.length} dy = P \int_0^{10} (10-y)(\frac{y+20}{2}) dy = \frac{3500}{3} P$  □

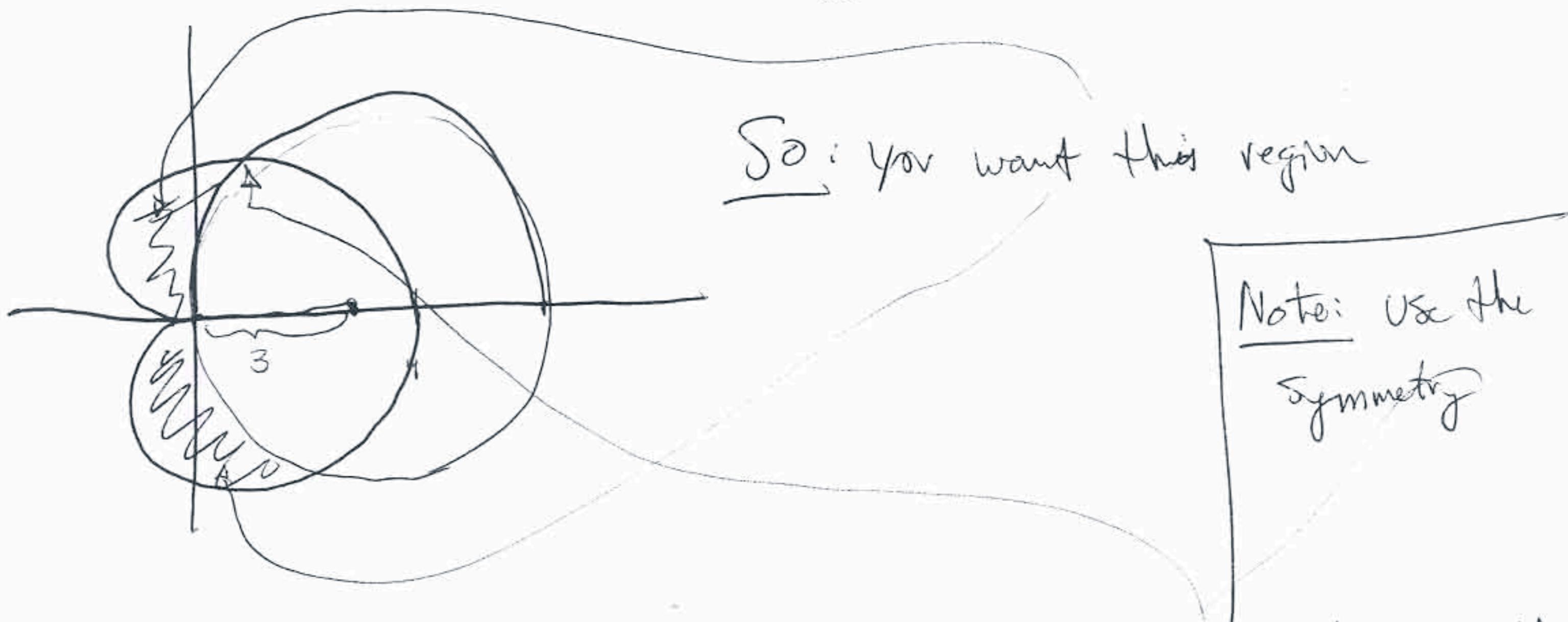
② It would help to draw the figure to get an idea...

$$r = 6 \cos \theta \text{ is a circle since } r \cdot r = r \cdot 6 \cos \theta = r^2 = 6r \cos(\theta)$$

$$\text{So } x^2 + y^2 = 6x \text{ and } x^2 - 6x + y^2 = x^2 - 6x + 9 + y^2 = 9$$

$$\text{So } (x-3)^2 + y^2 = 9 \text{ i.e. we have the circle}$$

Cardioid is as follows--



Now: first need the point of intersection here and using the

formulas we will be done.

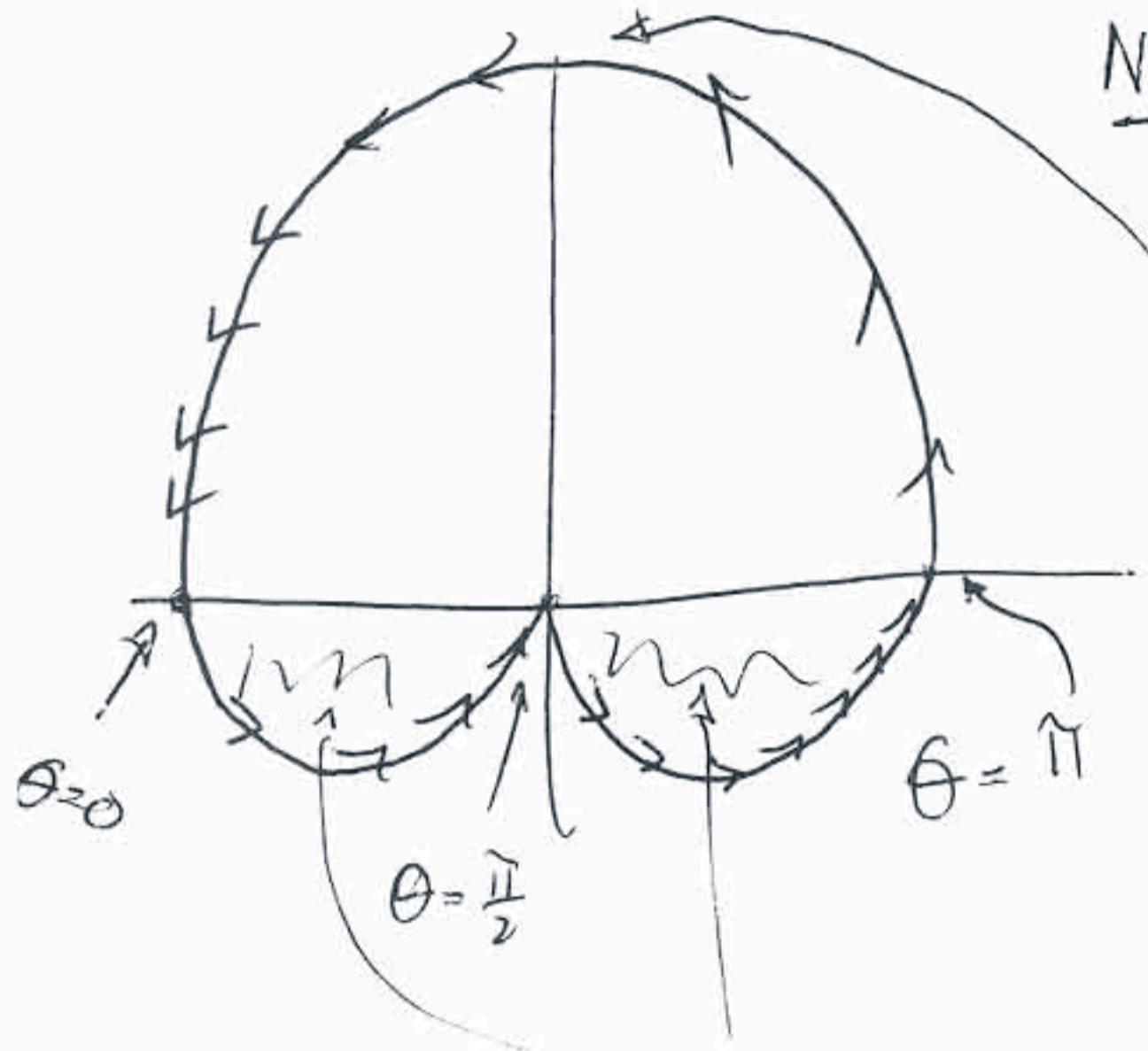
$$2 + 2 \cos \theta = 6 \cos \theta \Rightarrow 2 = 4 \cos \theta \Rightarrow \cos(\theta) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}.$$

$$\text{Then: Area} = 2 \left( \frac{1}{2} \int_{\frac{\pi}{3}}^{\pi} (2 + 2 \cos \theta)^2 d\theta - \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (6 \cos \theta)^2 d\theta \right)$$

And this gives an answer of  $\pi$ .

You MUST SHOW WORK, although I haven't.

③ Again; draw it.



Note: You move around in a strange way

Since  $\sin(\theta) - 1 \leq 0$  for  $0 \leq \theta \leq 2\pi$  ..

$$\theta = \frac{3\pi}{2}$$

You want this region, so  $2 \left( \frac{1}{2} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\sin(\theta) - 1)^2 d\theta \right) = \frac{3\pi}{4} - 2$ .

④ This is a partial fractions problem.

3 or 2 possibilities:  $\frac{A}{x^2+4} + \frac{B}{x+3} = \frac{x^2+13x+4}{(x^2+4)(x+3)}$

(would find  $B=1$  and this won't work.)

Another try with  $\frac{Ax}{x^2+4} + \frac{B}{x+3}$  won't work because it forces  $A=0$ .

But:  $\frac{Ax+B}{x^2+4} + \frac{C}{x+3}$  will probably work, and it does giving  $A=3$ ,  $B=4$ ,  $C=-2$ .

So we get

$$\int \frac{x^2+13x+4}{(x^2+4)(x+3)} dx = \int \frac{3x+4}{x^2+4} dx - 2 \int \frac{dx}{x+3}$$
$$= \frac{3}{2} \ln|x^2+4| + 2 \tan^{-1}\left(\frac{x}{2}\right) - 2 \ln|x+3| + C$$

End Review 3

## Review IV.

- ⑥ What is the alternating Series Convergence test?
- A  $a_{k+1} > a_{k+2} \geq 0$  if you can verify these 2 things you can  
 B  $\lim_{k \rightarrow \infty} a_k = 0$  conclude convergence.
- 

So: A must verify that  $\frac{(8(n+1)+5)}{(n+1)(2(n+1)+1)} \leq \frac{8n+5}{n(2n+1)}$

Note: This is a necessary step in solving the problem.

if you can show that

$$(8(n+1)+5)n(2n+1) \leq (8n+5)(n+1)(2(n+1)+1), \text{ then}$$

you are done... multiplying it all out in steps --

we get

$$\begin{aligned} (8(n+1)+5)n(2n+1) &= (8n+13)(2n^2+n) = 16n^3 + 26n^2 + 8n^2 + 13n \\ &= 16n^3 + 34n^2 + 13n \end{aligned}$$

$$(8n+5)(n+1)(2(n+1)+1) = (8n+5)(n+1)(2n+3)$$

$$= (8n+5)(2n^2 + 5n + 3) = 16n^3 + 50n^2 + 49n + 15$$

and so we compare them.

$$16n^3 + 34n^2 + 13n \leq 16n^3 + 50n^2 + 49n + 15 \quad ? \text{ for } n \geq 1 \quad ?$$

Yes, since  $0 \leq 16n^2 + 36n + 15$ . So A is verified.

B is easy ..

So we are done at this point. we only need to say

that both hypotheses of the a.s.c.t. are satisfied

so the series converges.

② If you guessed  $p=2$ , good guess.

You make the guess by noticing that if  $n$  is

very large,  $\frac{s_{n+1}}{n(n^2+1)} \approx \frac{s}{n^2}$  ..

So: Want  $C > 0$  st,  $\frac{s_{n+1}}{n(n^2+1)} \leq \frac{C}{n^2}$  for  $n \geq 1$ .

i.e. want  $C > 0$  s.t.  $n(8n+7) \leq C(n^2+1)$  and so

$$C \text{ s.t. } 0 \leq (C-8)n^2 - 7n + C.$$

Actually, we don't need  $\forall$  for  $n \geq 1$ , we only need if

$\exists n \geq N$  for some  $N$ , and so if we take

$$C=9, \text{ we get } 0 \leq n^2 - 7n + 9 \text{ for } \underline{n \geq 6}.$$

So  $\sum_{n \geq 6} \frac{8n+7}{n(n^2+1)} \leq \frac{9}{n^2}$ .

Now: Since  $\sum_{n=1}^{\infty} \frac{9}{n^2}$  converges by the p-test,

by comparison  $\sum_{n=1}^{\infty} \frac{8n+7}{n(n^2+1)}$  converges.  $\square$

③ Simple application of the ratio test.. For

$\sum_{n=1}^{\infty} a_n$ , the series converges if

$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$  and diverges if the limit is greater than 1.

Make sure to do your arithmetic & cancelation

property: Very few of you do & you turn an easy problem into a wrong answer.

So: we want to find  $c$  s.t. the series converges if  $-c < x < c$ ...

Apply the ratio test to the terms to get

$$\lim_{n \rightarrow \infty} \left| \frac{2^{n+1} (n+3) x^{n+1}}{5^{n+1} (3n+5)} \cdot \frac{5^n (3n+5)}{2^n (n+2) x^n} \right| = \lim_{n \rightarrow \infty} \left( \frac{2}{5} \left| \left( \frac{n+3}{n+2} \right) \left( \frac{3n+5}{3n+8} \right) x \right| \right) = \frac{2|x|}{5}.$$

And we want  $|x| < \frac{5}{2}$  so

$-\frac{5}{2} < x < \frac{5}{2}$ . But we aren't done since

we must check the endpoints...

if  $x = \frac{5}{2}$ , series is  $\sum_{n=0}^{\infty} \frac{n+2}{3n+5}$  which diverges. [why?]

Same for  $x = -\frac{5}{2}$ . So the exact interval is

$-\frac{5}{2} < x < \frac{5}{2}$   $\square$

⑨ Again: if you don't pay attention on these you will lose points...

$$\frac{d}{dx} (4+x)^{\frac{3}{2}} = \frac{3}{2}(4+x)^{\frac{1}{2}} \approx 3 + \frac{3}{8}x - \frac{3}{128}x^2 + \frac{3x^3}{1024} - \frac{15}{32768}x^4$$

so the Taylor poly for  $(4+x)^{\frac{1}{2}}$  is

$$\frac{2}{3} \left( 3 + \frac{3}{8}x - \frac{3}{128}x^2 + \frac{3x^3}{1024} - \frac{15}{32768}x^4 \right) \dots$$

$$\int (4+x)^{\frac{3}{2}} dx = \frac{2}{5} (4+x)^{\frac{5}{2}} \approx 8x + \frac{3}{2}x^2 + \frac{20}{16}x^3 - \frac{x^4}{4.128} + \frac{3x^5}{5.4096} - \frac{3.6}{32768}x^5 + C$$

$$\text{So } (4+x)^{\frac{5}{2}} \approx \frac{5}{2} \left( 8x + \frac{3}{2}x^2 + \frac{x^3}{16} - \frac{x^4}{4.128} + \frac{3x^5}{5.4096} - \frac{18}{32768}x^5 + C \right)$$

Where  $\frac{5}{2}C = (4+0)^{\frac{5}{2}} = 2^5, \square$

$$C = \frac{2^6}{5}$$

$$\text{So } (4+x)^{\frac{5}{2}} \approx \frac{5}{2} \left( 8x + \frac{3}{2}x^2 + \cancel{\frac{x^3}{16}} - \cancel{\frac{x^4}{4.128}} + \cancel{\frac{3x^5}{5.4096}} - \cancel{\frac{18x^5}{32768}} + \frac{2^6}{5} \right) \quad \square$$

End of rev IV

## Review I

① For a line you need a direction vector and a point.

The tangent line at  $(17, 2)$  has direction vector  $\vec{r}'(t_0)$  where  $t_0$  is such that

$$\vec{r}(t_0) = (17, 2).$$

So:  $17 = 3t^2 + 5 \Rightarrow 3t^2 = 12 \nmid t_0 = \pm 2.$

$t_0 = 2$  works for  $2(2)^2 - 6 = 2..$

so,  $t_0 = 2.$

$$\vec{r}'(t) = 6t\vec{i} + (4t - 3)\vec{j}, \text{ at } t=2 \text{ we get}$$

$$\vec{r}'(2) = 12\vec{i} + 5\vec{j} \text{ and the tangent line is}$$

$$\vec{R}(t) = 17\vec{i} + 2\vec{j} + t(12\vec{i} + 5\vec{j}).$$

Easy, Easy, Easy) if you know what you're doing!

② MUST IDENTIFY THAT the Series is alternating, so you can use the a.s.e.t. to find a bound for the remainder,  $\sum_{k=9}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = R_8$

Remember  $\sum_{k=0}^{\infty} a_k = S_N + R_N$  where

$$R_N = \sum_{k=N+1}^{\infty} a_k \quad \& \quad S_N = \sum_{k=1}^N a_k$$

a.s.e.t. says that  $|R_N| \leq \left| \frac{x^{2(N+1)+1}}{2(N+1)+1} \right|$ , and since  $N=8$ ,

$$|R_N| \leq \left| \frac{x^{2 \cdot 9 + 1}}{2 \cdot 9 + 1} \right| = \frac{|x|^{19}}{19}, \text{ and we take}$$

$$\frac{|x|}{19} < 10^{-6} \text{, i.e. } |x| < \sqrt[19]{10^{-6}} \approx .5643$$

③  $|x| \leq .8$ , and we have an alternating series.

$$|R_N| \leq \left| \frac{x^{2(N+1)+1}}{(2(N+1)+1)!} \right| \leq \frac{(.8)^{2N+3}}{(2N+3)!}$$

and we want the smallest  $N$  s.t.

$$\frac{(.8)^{2N+3}}{(2N+3)!} < 10^{-8}$$

Note: There is no way to undo this so, test values ..

1?, no, 2?, no, 3?, no, 4?, yes

so  $N=4$ .

④  $\theta = \cos^{-1} \left( \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} \right) = \cos^{-1} \left( \frac{6}{5\sqrt{30}} \right) \approx 1.375$



$$\text{Proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \vec{a} = \frac{6}{38} (3_i - 5_j + 2k)$$

Fewer VI

① Tricky to trace these, ...  $\pi \leq \theta \leq 2\pi$

② Pick one point,  $(5, 0, -3)$  and let

$$\vec{V} = (2, -4, 7) - (5, 0, -3) = (-3, -4, 10)$$

$$\vec{W} = (1, 4, -6) - (5, 0, -3) = (-4, 4, -3)$$

$$\vec{N} = \vec{V} \times \vec{W} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -4 & 10 \\ -4 & 4 & -3 \end{vmatrix} = \begin{vmatrix} -4 & 10 \\ 4 & -3 \end{vmatrix} \vec{i} - \begin{vmatrix} -3 & 10 \\ -4 & -3 \end{vmatrix} \vec{j} + \begin{vmatrix} -3 & -4 \\ -4 & 4 \end{vmatrix} \vec{k}$$

$$= -28\vec{i} - 49\vec{j} - 28\vec{k}$$

Then: The equation will be

$$\vec{N} \cdot (\vec{x} - (5, 0, -3)) = 0,$$

$$\text{i.e. } (-28, -49, -28) \cdot (x-5, y, z+3) = 0$$

$$\text{and } 28x + 49y + 28z = 56 \quad \square$$

③ For any geometric Series, we have that it is

$$a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{n-1}$$

To find  $r$ , just take the  $2^{\text{nd}}$  term & divide it by the  $1^{\text{st}}$ :

$$r = \frac{96}{64} = \frac{4 \cdot 24}{4 \cdot 16} = \frac{16 \cdot 6}{4 \cdot 16} = \frac{3}{2}.$$

So Series is  $64 + 64 \cdot \frac{3}{2} + 64 \left(\frac{3}{2}\right)^2 + \dots + 64 \left(\frac{3}{2}\right)^{n-1}$

$$64 \left(\frac{3}{2}\right)^{n-1} = \frac{59049}{16} \quad \text{and} \quad \text{So}$$

$$\left(\frac{3}{2}\right)^{n-1} = \frac{59049}{64 \cdot 16} \quad \text{and by testing values we have}$$

$$n-1 = 10, \quad \text{i.e.} \quad n = 11.$$

④ Easy to mess these up, but be patient &  
pay attention.

Set  $4t - 2 = 3s + 14$ , then  $t = \frac{3s + 16}{4}$

Substitute into  $-2t = 5s + 5$ , i.e.

$$-2\left(\frac{3s + 16}{4}\right) = 5s + 5, \text{ and so } 3s + 16 = -10s - 10,$$

$$13s = -26, \text{ i.e. } s = -2, \text{ so } t = \frac{-6 + 16}{4} = \frac{10}{4} = \frac{5}{2}.$$

Now: must plug in  $s$  &  $t$  to the last coordinates  
to see if they work...

$$\frac{6 \cdot 5}{2} - 15 = 0, \text{ & } -2(-2) - 4 = 0, \text{ so } \underline{\text{yes}} \text{ the}$$

lines intersect at the point

$$\left(\frac{6 \cdot 5}{2} - 2, -2\left(\frac{5}{2}\right), 0\right) = (8, -5, 0),$$

Very simple, so don't lose points on this one.

⑤ You must pay attention to the sign of the entries to figure out exactly which angle you start with if you take the radius  $r$  to be positive.

$(+, +) \Rightarrow 1^{\text{st}}$  quadrant

$(-, +) \Rightarrow 2^{\text{nd}}$  quadrant

$(-, -) \Rightarrow 3^{\text{rd}}$  quadrant

$(+, -) \Rightarrow 4^{\text{th}}$  quadrant

So:  $r^2 = x^2 + y^2$  .....

a)  $r = \sqrt{(-4\sqrt{3})^2 + 4^2} = \sqrt{3 \cdot 16 + 16} = \sqrt{4 \cdot 16} = 8$

$$\cos(\theta) = \frac{x}{r}, \quad \sin(\theta) = \frac{y}{r}, \quad \text{so } \cos(\theta) = \frac{-4\sqrt{3}}{8} = -\frac{\sqrt{3}}{2}$$

~~cosine~~ we know from the unit triangle that we



$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$
$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$	$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

must have something like  $\frac{\pi}{6}$  in the 2<sup>nd</sup> quadrant, only change is  $\frac{5\pi}{6}$

and we have  $(8, \frac{5\pi}{6})$  is one coordinate ...

If we add or subtract  $2\pi$  to the angle, we get the

same point, So  $(8, \frac{5\pi}{6} - 2\pi) = (8, -\frac{7\pi}{6})$  is another representation..

If we add or subtract  $\pi$ , & take the negative of the radius, we also get it so  $(-8, \frac{5\pi}{6} \pm \pi)$  works too..

b) Same deal here.  $(6\sqrt{2}, \frac{5\pi}{4})$   $(-6\sqrt{2}, \frac{\pi}{4})$   
 $(6\sqrt{2}, -\frac{3\pi}{4})$   $(-6\sqrt{2}, -\frac{7\pi}{4})$

⑤ Just like with geometric series -- an arithmetic one has its own formula & working ..

$$a_1 + (d+a_1) + (2d+a_1) + (3d+a_1) + \dots + ((n-1)d+a_1)$$

To find  $d$ , just Subtract the 1<sup>st</sup> term from the 2<sup>nd</sup>.

$$-7 - (-13) = -7 + 13 = 6 \therefore d = 6.$$

Then: the last term is  $1199 = -13 + (n-1) \cdot 6$ ,  
and  $n = 203 \dots$

Finally the sum is  $\frac{n}{2}(a_1 + a_m)$  so we have

$$S_{203} = \frac{203}{2}(-13 + 1199) = 120,379.$$

DONE.

Good luck to everyone & I  
wish you all Success.