

**Brief answers to assigned even numbered problems that were not  
to be turned in**

**Section 2.2**

2. At point  $(x_0, x_0^2)$  on the curve the slope is  $2x_0$ . The point-slope equation of the tangent line to the curve at this point is

$$y - x_0^2 = 2x_0(x - x_0).$$

The  $x$ -intercept is obtained by setting  $y = 0$  in this equation and solving for  $x$ :  $x = x_0/2$ .

**Section 2.3**

**24.**

$$\begin{aligned} f(x + \Delta x) - f(x) &= [3(x + \Delta x)^2 + 4(x + \Delta x) - 5] - [3x^3 + 4x - 5] \\ &= 6x\Delta x + (\Delta x)^2 + 4\Delta x. \\ \frac{f(x + \Delta x) - f(x)}{\Delta x} &= 6x + \Delta x + 4 \\ f'(x) &= \lim_{\Delta x \rightarrow 0} (6x + \Delta x + 4) = 6x + 4. \end{aligned}$$

**40.**  $f'(x) = 3x^2 - 3$ . Therefore

$$f'(x) = 0 \iff 3x^2 = 3 \iff x = \pm 1.$$

Now  $f(1) = -2$  (low), and  $f(-1) = 2$  (high).

**Section 2.4**

**10.**  $s(t) = 6t^2 + 2t$ ,  $s'(t) = 12t + 2$

**10a.**  $\frac{s(6)-s(3)}{6-3} = 56$  ft/sec

**10b.**  $s'(3) = 38$  ft/sec

**106.**  $s'(6) = 74$  ft/sec

## Section 2.5

**4.** limit doesn't exist

**6.** 2

**18c.** 5/2

**20c.**

$$\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos^2 x} = \lim_{x \rightarrow 0} \frac{x^2}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1 \times 1 = 1.$$

**20f.**

$$\lim_{\theta \rightarrow 0} \frac{\theta^2 - 2 \sin \theta}{\theta} = \lim_{\theta \rightarrow 0} \theta - 2 \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 0 - 2 \times 1 = -2.$$

**20g.** Set  $h = 2x$ .

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{3x^2 + 4x}{\sin 2x} &= \lim_{h \rightarrow 0} \frac{\frac{3}{4}h^2 + 2h}{\sin h} = \frac{3}{4} \lim_{h \rightarrow 0} h \times \lim_{h \rightarrow 0} \frac{h}{\sin h} + 2 \lim_{h \rightarrow 0} \frac{h}{\sin h} \\ &= \frac{3}{4} \times 0 \times 1 - 2 \times 1 = -2. \end{aligned}$$

## Section 18.1

**6a.** horizontal circle with radius 2, center at  $(0, 0, 3)$ .

**6b.** parabola in the plane  $x = 4$ .

**6c.** vertical line  $(5, 5, z)$ .

**6d.** parabola in the  $x - z$  plane.

**8.** plane  $-5x + y + 3z = -12$ .

**14a.**  $(x - 2)^2 + (y - 3)^2 + (z - 1)^2 = 21$ .

**14b.**  $(x + 3)^2 + (y - 4)^2 + (z + 2)^2 = 43$ .

## Section 18.2

**8a.**  $c = 2/5$

**8b.**  $c = -2$

**14.** Let  $\mathbf{N} = a \mathbf{i} + b \mathbf{j}$ . I claim that  $\mathbf{N}$  is perpendicular to the line  $L$  :  $ax + by + c = 0$ . Indeed if  $P_1(x_1, y_1)$ ,  $P_2(x_2, y_2)$  are two distinct points on  $L$ , so that

$$ax_1 + by_1 + c = 0, \quad ax_2 + by_2 + c = 0,$$

then, subtracting one equation from the other we have  $a(x_2 - x_1) + b(y_2 - y_1) = 0$  or, in terms of the dot product,  $\mathbf{N} \cdot \mathbf{P}_2 \mathbf{P}_1 = 0$ . Now  $\mathbf{P}_2 \mathbf{P}_1$  is pointed along  $L$  and it is perpendicular to  $\mathbf{N}$ , so  $\mathbf{N}$  is perpendicular to  $L$ . We are given the point  $P_0(x_0, y_0)$ . Consider the right triangle  $\triangle P_0 Q P_1$  where  $Q$  is the orthogonal projection of  $P_0$  on  $L$ , i.e.,  $Q$  is the point such that the line through  $P_0$  in the direction of  $\mathbf{N}$  intersects  $L$ . Then, from trigonometry, the distance  $D$  from  $P_0$  to  $L$  is the absolute value of  $\|\mathbf{P}_0 \mathbf{P}_1\| \cos \theta$  where  $\theta$  is the angle between  $\mathbf{N}$  and  $\mathbf{P}_0 \mathbf{P}_1$ . From the basic geometrical property of the dot product we have

$$|\mathbf{N} \cdot \mathbf{P}_0 \mathbf{P}_1| = \|\mathbf{N}\| \|\mathbf{P}_0 \mathbf{P}_1\| |\cos \theta| = \|\mathbf{N}\| D.$$

Thus

$$D = \frac{|\mathbf{N} \cdot \mathbf{P}_0 \mathbf{P}_1|}{\|\mathbf{N}\|}.$$

Now  $\|\mathbf{N}\| = \sqrt{a^2 + b^2}$  and

$$\mathbf{N} \cdot \mathbf{P}_0 \mathbf{P}_1 = a(x_0 - x_1) + b(y_0 - y_1) = ax_0 + by_0 + c,$$

since  $ax_1 + by_1 + c = 0$ . We conclude that

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$

## Section 18.4

**4a.**  $\frac{x-3}{4} = \frac{y}{-3} = \frac{z+2}{5}$ .

**4b.**  $\frac{x-3}{7} = \frac{y}{2} = \frac{z+2}{-3}$ .

**4c.**  $\frac{x-3}{1} = \frac{y}{0} = \frac{z+2}{0}$ .

**8.**  $(0, 2, -2), (3, 0, -1), (6, -2, 0)$ .

**20.**  $t = -3/2$ .

- 22.** Both lines pass through the distinct points  $(4, 6, -9)$  and  $(1, 2, 3)$ , so they must be the same line.

## Section 2.6

- 40.** Suppose the distance from  $W_1$  to  $W_2$  is  $L > 0$ . Then we can parametrize the points on the straight horizontal string touching the walls by  $x$ , where  $0 \leq x \leq L$ . The distance of the point with coordinate  $x$  on the string to wall  $W_1$  is  $x$ , and the distance to wall  $W_2$  is  $L - x$ . When the string is wadded up, let the distance from the same point with coordinate  $x$  on the string to wall  $W_1$  be  $D(x)$ , so that the distance to wall  $W_2$  is  $L - D(x)$ . Since the wadded string doesn't touch the walls we see that  $D(x)$  is a continuous function such that  $0 < D(x) < L$  for all  $x \in [0, L]$ . From problem 39 there is a point  $c$  in  $(0, L)$  such that  $D(c) = c$ . This finishes the proof.

### Section 3.1

**4a.**  $x^3$ .

**4b.**  $\frac{4}{3}x^3$ .

**4c.**  $x^3 + x^2 - 5x$ .

**6.**  $(1, -4), (2, 18)$ .

**18.**  $y = 3x + 2$ .

**22.**  $y = 2x - 1$ .

### Section 3.2

**8.**  $(4x^3)(x^4 - 1) + (x^4 + 1)(4x^3) = 8x^7$ .

**10.**  $-\frac{2x}{(x^2+2)^2}$ .

**14.**  $\frac{-x^6-16x^3+8}{(x^3+2)^2}$ .

### Section 3.3

**14.**  $28(3 - 4x)^{-8}$ .

**24.**  $5(x^3 - 7x)^4(3x^2 - 7)$ .

**26.**  $-\frac{5(3x^2-5)}{(x^3-5x+1)^6}$ .

**36.**  $\frac{(4t^3-20t)(t^2-6)^2+2(t^4-10t^2)(t^2-6)(2t)}{(t^2-6)^4}$ .

**46a.**  $(x^2 - 1)^2$ .

**46b.**  $\frac{4}{3}(x^2 - 1)^3$ .

**46c.**  $(x^3 - 2)^2$ .

**46d.**  $(x^3 - 2)^3$ .

### Section 3.4

**2.**  $-4x^4 \sin(x^5 + 1)$ .

**6.**  $24(4x - 1)^2 \sin(4x - 1)^3 \cos(4x - 1)^3$ .

**22.**  $3 \cos^3 x - 2 \cos x$ .

**24.**  $\sin x + x \cos x$ .

**30.** 0.

**34.**

$$\frac{2\pi}{3} + 2\pi n, \quad \frac{4\pi}{3} + 2\pi n, \quad n = 0 \pm 1, \pm 2, \dots$$

### Section 3.5

**2.**  $y' = \frac{-y^2 + 2xy - 2x}{2xy - x^2 + 4y}$

**4.**  $y' = \frac{-4x^3y^3 + 3y}{3x^4y^2 - 3x}$

**36.**  $y' = \frac{1}{1+x^2}$

**42.**  $-\frac{\sin x}{x^{\frac{1}{2}}} - \frac{\cos x}{2x^{\frac{3}{2}}}$

## Section 3.6

**2a.**  $y'' = \frac{2}{(1-x)^3}$

**2b.**  $y'' = 2 - \frac{6}{x^4}$

**2c.**  $y'' = \frac{4}{(1+x)^3}$

**2d.**  $y'' = 6x - \frac{12}{x^5}$

**2e.** 0

**4a.**  $y'' = -\frac{b^2}{a^2y} - \frac{b^4x^2}{a^4y^3}$

**4b.**  $y'' = \frac{4p^2}{y^3}$

**4c.**  $y'' = \frac{1}{2x} + \frac{\frac{1}{2}}{2x^{\frac{3}{2}}}$

**4d.**  $y'' = -\frac{2x}{y^2} - \frac{2x^4}{y^5}$

**4e.**  $y'' = -\frac{3x^2}{y^3} - \frac{3x^6}{y^7}$

**8a.** 0

**8b.** 22!

## Section 17.1

4.  $(\frac{x}{3})^2 + (\frac{y}{2})^2 = 1$

8.  $x^2 = (1 - y)^3$

## Section 4.1

22. Example:  $f(x) = -x$ , domain:  $[-2, 1]$

28a. max:  $\sin 1$ , min:  $-\sin 1$

28b. max: 1, min:  $\cos 1$

## Section 4.2

4. inflection pt.:  $x = -\frac{1}{2}$ , conc. up:  $x > -\frac{1}{2}$ , conc. down:  $x < -\frac{1}{2}$

16. No

18.  $y'' = -\frac{a^2}{y^3}$

## Section 4.3

2. 6, 12

4. Let  $A$  be the given area of the rectangle, and let  $x$  be its width. Then the length of the rectangle is  $A/x$  and the perimeter is

$$P(x) = 2x + \frac{2A}{x}, \quad 0 < x.$$

The derivative of the perimeter function is

$$P'(x) = 2\left(1 - \frac{A}{x^2}\right),$$

and this is negative for  $0 < x < \sqrt{A}$ , zero for  $x = \sqrt{A}$ , and positive for  $\sqrt{A} < x$ . Thus the minimum area occurs for  $x = \sqrt{A}$ , the case with equal width and length.

10.  $50' \times 70'$

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40a.  $y' = -\frac{4x^3+2y^3}{6xy^2+8y^3}$

40b.  $y' = \frac{2x^2+y}{x-x^2}$

40c.  $y' = -\frac{4x}{y(x^2-2)^2}$

40d.  $y' = \frac{4x^3-4x^3y^4}{4y^3x^4-4y^3}$

40e.  $y' = \frac{\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}}}{2+\frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}}$

### Section 4.4

2. base/height = 2

### Section 4.5

2a.  $-\frac{8}{25}$  ft/h

2b.  $-\frac{16\pi}{5}$  ft<sup>2</sup>/h

### Section 17.3

14a.  $(600 + 30\sqrt{2}) \mathbf{i}_{east} + 30\sqrt{2} \mathbf{j}_{north}$  m/h

14b.  $330\sqrt{2} \mathbf{i}_{east} - 270\sqrt{2} \mathbf{j}_{north}$  m/h

### Section 17.4

6.  $\frac{d\mathbf{R}}{dt} = 2t \mathbf{i} + 3t^2 \mathbf{j}$ ,  $\frac{d^2\mathbf{R}}{dt^2} = 2 \mathbf{i} + 6t \mathbf{j}$ ,  $\left|\left|\frac{d\mathbf{R}}{dt}\right|\right| = \sqrt{4t^2 + 9t^4}$

10.  $\mathbf{R} = a \cos kt \mathbf{i} + b \sin kt \mathbf{j}$ ,  $\frac{d\mathbf{R}}{dt} = -ak \sin kt \mathbf{i} + bk \cos kt \mathbf{j}$ ,  $\frac{d^2\mathbf{R}}{dt^2} = -ak^2 \cos kt \mathbf{i} - bk^2 \sin kt \mathbf{j} = -k^2 \mathbf{R}$ , spring attached to origin, with spring constant  $k^2$ , i.e., the force is directed toward the origin with magnitude proportional to the distance from the origin.

### Section 5.4

10.  $y = \sqrt{x^2 + 5}$

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**34.** The family of ellipses  $y^2 + \frac{x^2}{2} = k^2$

### Section 6.3

**2a.**  $\sum_{k=1}^4 3^k$

**2b.**  $\sum_{k=0}^5 (-1)^k (2k+3)$

**2d.**  $\sum_{k=0}^{200} 2^k$

**2e.**  $\sum_{k=1}^5 a^{5+k}$

**2g.**  $\sum_{k=1}^{16} k^2$

### Section 6.6

**4.** 16

**10.**  $\frac{63}{4}$

**22.**  $x^n$  isn't continuous on  $[-1, 1]$

**38.**  $\frac{41}{6}$

### Section 6.7

**2a.**  $\frac{16}{3}$

**2b.**  $\frac{1}{6}a^2$

**4.**  $\frac{256}{15}$

**16a.**  $\frac{1}{x+2}$

$$16d. -\frac{1}{1+x^4}$$

$$16e. \frac{2x}{\sqrt{x^2 + \sqrt{x^2 + 1}}}$$

### Section 8.2

$$4b. 32$$

$$4d. 8$$

### Section 8.3

$$4. (2x - 2x^3)e^{-x^2}$$

$$6. ex^{e-1} + e^x$$

$$12. -\frac{e^{-x^2}}{2} + C$$

### Section 8.4

$$4b. \frac{1}{x^2-1}$$

$$4d. \frac{2}{2x+1} - \frac{1}{x+2}$$

$$4g. \frac{3}{x}$$

$$4k. \frac{12}{3x-7} + \frac{6}{2x+5}$$

$$16. \text{ max at } (e, \frac{1}{e}), \text{ inflection point at } (e^{\frac{3}{2}}, \frac{3}{2}e^{-\frac{3}{2}})$$

$$18. \frac{1}{3} ((x+1)(x-2)(2x+7))^{\frac{1}{3}} \left( \frac{1}{x+1} + \frac{1}{x-2} + \frac{2}{2x+7} \right)$$

**22b.** We know that  $\lim_{x \rightarrow \infty} x^{\frac{1}{x}} = 1$ . Taking the logarithm we then have  $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \ln 1 = 0$ . Substitute  $x = \ln y$  in the last limit to get  $\lim_{y \rightarrow \infty} \frac{\ln(\ln y)}{\ln y} = 0$ . Thus

$$\lim_{y \rightarrow \infty} \frac{\ln(\ln y)}{y} = \lim_{y \rightarrow \infty} \left[ \frac{\ln y}{y} \frac{\ln(\ln y)}{\ln y} \right] = \lim_{y \rightarrow \infty} \frac{\ln y}{y} \cdot \lim_{y \rightarrow \infty} \frac{\ln(\ln y)}{\ln y} = 0 \cdot 0 = 0.$$

Now  $\frac{\ln(\ln y)}{y} = \ln[(\ln y)^{\frac{1}{y}}]$ . Exponentiating, we have  $\lim_{y \rightarrow \infty} (\ln y)^{\frac{1}{y}} = 1$ , so

$$\lim_{n \rightarrow \infty} (n \ln n)^{\frac{1}{n}} = \lim_{n \rightarrow \infty} n^{\frac{1}{n}} \cdot \lim_{n \rightarrow \infty} (\ln n)^{\frac{1}{n}} = 1 \cdot 1 = 1.$$

## Section 7.2

**6.** 4

**10.**

$$A = \int_{-5}^{-2} (x+5) \, dx + \int_{-2}^0 3 \, dx + \int_0^1 (1 - \sqrt{x}) \, dx + \int_0^4 (-\sqrt{x} + 2) \, dx = \frac{27}{2}.$$

**18.** 1

**24.**  $16^{\frac{1}{3}}$

**26.**  $A(b) = 1 - \frac{1}{b} \rightarrow 1$  as  $b \rightarrow \infty$ .

## Section 7.3

**6.**  $V_1 = 4\pi, \quad V_2 = 4\pi$

**18.**  $A(b) = 3b^{\frac{1}{3}} - 1 \rightarrow +\infty$  as  $b \rightarrow +\infty$ .  $V(b) = 3\pi(1 - 3b^{-\frac{1}{3}}) \rightarrow 3\pi$  as  $b \rightarrow +\infty$ .

### Section 7.4

6.  $\frac{16\pi}{3}$

8.  $\frac{\pi}{8}$

10.  $\frac{28\pi}{3}$

### Section 7.5

2.  $\frac{123}{32}$

6.  $\frac{495}{8}$

8. 21

### Section 7.6

2.  $3\pi$

6.  $8\pi(9 - \sqrt{3})$

8.  $\frac{56\pi\sqrt{3}}{5}$

### Section 7.7

2. 300 in.-lbs.

6. 20,000 ft.-lbs.

**14.** If the force is  $F(x) = k/x^2$ , the work done in the first case is  $k/2$  and in the second case  $k(1/b - 1/a)$ .

**16.** 64 lbs., 25 lbs., 80,000 mi.-lbs.

### Section 7.8

**4.** 297,000 lbs.

**6.** 18,750 lbs.

**14.**  $27w_1 + 45w_2$

### Section 8.6

**10.**  $\frac{500}{3} \ln 10$  days

### Section 9.3

**26.**  $\sqrt{2} - 1$

**30.**  $\pi/3$

### Section 9.4

**24.**  $\pi (\sqrt{3} - \pi/3)$

## Section 9.5

10.

$$\arctan x + \frac{x}{1+x^2} - \frac{x}{\sqrt{1+x^2}}$$

12.  $\sqrt{1-x^2}$

20.  $\frac{1}{4} \arctan(2x^2) + c$

22.

$$\frac{3}{16} \arcsin\left(\frac{4x}{3}\right) + c$$

28 10/136 radians/sec.

## Section 9.6

2. Show  $\frac{dv}{dt} = -2a^2x\frac{dx}{dt} = -2a^2xv$ . Thus  $\frac{dv}{dt} = 0$  if and only if  $x = 0$  or  $v = 0$ . Examine these critical points.

6. Weight of buoy is  $mg = \frac{2\pi}{3}r^2\rho$ . Force term is  $F(y) = \pi\rho(r^2y - y^3/3) \approx \pi\rho r^2y$  for  $y$  small. Thus,  $-\pi\rho r^2y = my''$  and vibration equation is  $y'' + \frac{3g}{2}y = 0$ , so frequency of solutions is  $f = \sqrt{3g/2}/2\pi$ .

## Section 10.2

4.  $\exp(\sin x) + C$

18.  $\ln(x^2 + x + 2) + C$

## Section 10.3

6.  $(\sin^3 x)/3 - 2(\sin^5 x)/5 + (\sin^7 x)/7 + C$

8.  $1/12$

### Section 10.4

12.

$$\frac{x^2}{2} - \frac{a^2}{2} \ln(a^2 + x^2) + C$$

16.

$$a^5 \left( \frac{\sec^5 \theta}{5} - \frac{\sec^3 \theta}{3} \right) + C$$

18.

$$\frac{-x}{a^2 \sqrt{x^2 - a^2}} + C$$

### Section 10.5

2.

$$\arcsin \left( \frac{x-2}{3} \right) + C$$

4.

$$\frac{2\sqrt{3}}{3} \arctan \left( \frac{2\sqrt{3}}{3} \left( x - \frac{1}{2} \right) \right) + C$$

6.

$$-\sqrt{5 + 4x - x^2} + 5 \arcsin \left( \frac{x-2}{3} \right) + C$$

### Section 10.6

**6.**

$$\ln |x(x-2)^3(x-3)^5| + C$$

**10.**

$$\frac{3}{x} + \ln(x^2(x-3)^4) + C$$

**16.**

$$x + \frac{1}{x+1} + C$$

**20.**

$$\frac{x^2}{2} - 3x + \ln(\sqrt{x^2 + 1}) + C$$

**24.**

$$-\ln(1 + e^{-x}) + C$$

### Section 10.7

**8.**

$$x \arcsin x + \sqrt{1-x^2} + C$$

**12.**

$$-\frac{\arctan x}{x} + \ln \left| \frac{x}{\sqrt{1+x^2}} \right| + C$$

### Section 10.8

**26.**

$$(x+5)^{4/3} \left( \frac{4x-45}{28} \right) + C$$

**96.**

$$\frac{1}{3} \ln \left| \sqrt{9x^2 + 12x - 5} + 3x + 2 \right| + C$$

**102.**

$$\left(1 + \sqrt{1 + \sqrt{x}}\right)^{5/2} \left(\frac{488}{315} - \frac{104}{63}\sqrt{1 + \sqrt{x}} + \frac{8}{9}\sqrt{x}\right) + C$$

## Section 11.2

**16.** If we take the density  $\rho = 1$  then

$$M = \frac{1}{2}ac, \quad M_y = \frac{1}{6}ac(a+b), \quad M_x = \frac{1}{6}ac^2.$$

Thus

$$\bar{x} = \frac{M_y}{M} = \frac{1}{3}(a+b), \quad \bar{y} = \frac{1}{3}c.$$

The line through the vertex  $(b, c)$  to the midpoint  $(a/2, 0)$  of the opposite side has the equation

$$y = \frac{c}{2b-a}(2x-a)$$

and, clearly, the point  $(\bar{x}, \bar{y})$  lies on this line and is  $2/3$  of the way from  $(b, c)$  to  $(a/2, 0)$ . Since our labeling of coordinates was arbitrary, it is clear that the centroid lies on all 3 medians, hence they intersect at a common point  $2/3$  of the way from each median to the midpoint of the opposite side. Physical intuition suggests, that since the triangle balances when supported at the centroid, any axis through a vertex and the centroid will also provide balance, i.e., there will be zero torque about the axis. This is precisely the case if the axis goes through the midpoint of the opposite side, for then the torque produced by each thin strip parallel to the opposite side is zero, so that the total torque is zero.

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**4.** We have

$$x = \frac{1-t^2}{1+t^2}, \quad y = \frac{2t}{1+t^2}.$$

Solving for  $t^2$  in the first equation we find  $t^2 = (1-x)/(1+x)$ , so  $1+t^2 = 2/(1+x)$ . Substituting this result in the equation for  $y$  we get

$$t = \frac{1}{2}y(1+t^2) = \frac{1}{2}\left(\frac{2y}{1+x}\right) = \frac{y}{1+x}.$$

Thus there is a unique  $t$  for each  $(x, y)$  on the circle such that  $x \neq -1$ .

If  $x$  and  $y$  are rational numbers, then so is  $t = y/(1+x)$ .

**6.** We have

$$\cos \theta = \frac{1-t^2}{1+t^2}, \quad \sin \theta = \frac{2t}{1+t^2}.$$

Thus

$$d\sin \theta = \cos \theta \, d\theta = \left(\frac{2}{1+t^2} - \frac{2t(2t)}{(1+t^2)^2}\right) dt = \frac{2(1-t^2)}{(1+t^2)^2} dt.$$

Using the expression for  $\cos \theta$  above and solving for  $d\theta$  we find  $d\theta = 2 dt/(1+t^2)$ .

## Section 12.2

**6.** 7

**8.** 3

## Section 12.3

**10.** 1

**24.** 1

**30.** 1

**36.** 1

### Section 12.4

2.  $1/2$

8.  $1/2$

**24a.** converges

**24c.** diverges

**24d.** converges

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**118.** 8

### Section 13.2

**14.**

$$x_{n+1}(1 + x_n) = (1 + x_n) + 1$$

so as  $n \rightarrow \infty$  we have  $L(1 + L) = 1 + L + 1$  or  $L^2 = 2$ . Since  $x_1 = 1$  we have  $x_{n+1} > 1$  for all  $n \geq 1$  so  $L$  must be positive. Therefore,  $L = \sqrt{2}$ .

### Section 13.3

**2b.**  $27/2$

**2i.**  $(75 + 40\sqrt{3})/11$

**10a.**  $3/2$

**12a.**

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n+1}$$

**12b.**  $n/(2n+1) \rightarrow 1 \neq 0$  as  $n \rightarrow \infty$ . diverges

### Section 13.4

**2d.** first term converges by comparison with  $\sum 1/n^2$ , second term is a convergent example.

**2f.** This is  $1/\ln 2$  times the harmonic series. diverges

**2g.** converges since  $\sum 1/n^2$  converges

**2h.** all terms are 0. converges

**2i.** sum starts out as

$$1 + 1/\sqrt{2} + 0 - 1/\sqrt{2} - 1 - 1/\sqrt{2} + 0 + 1/\sqrt{2}$$

and then repeats. diverges

**6a.** telescopes.  $S_N = \arctan(N+1) - \arctan(1)$ .

$$\lim_{N \rightarrow \infty} S_N = \lim_{N \rightarrow \infty} (\arctan(N+1) - \arctan(1)) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

**6b.**

$$\ln\left(1 + \frac{1}{n}\right) = \ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln n$$

telescopes.

$$S_N = \ln(N+1) - \ln 1 \rightarrow +\infty$$

as  $N \rightarrow +\infty$

## Section 13.5

2. Converges. Compare with  $\sum 1/n^2$
6. Compare with  $\sum 1/2^n$ . Converges.
18. Compare with  $\sum 1/n^{121/120}$ . Diverges.

## Section 13.7

6. Converges.
8. Diverges.

## Section 13.8

4. Conditional convergence.
6. Diverges.
8. Conditional convergence.
26. Shown in class that

$$s_1 \geq s_3 \geq \cdots \geq s_{2m-1} \geq s_{2m+1} \geq s \geq s_{2m} \geq s_{2m-2} \geq \cdots \geq s_4 \geq s_2$$

for all integers  $m$ . Thus we have two cases:

1.  $s_{2m} < s < s_{2m+1}$  and  $s_{2m+1} - s_{2m} = a_{2m+1}$  so that  $|s_{2m} - s| \leq a_{2m+1}$ .
2.  $s_{2m+2} < s < s_{2m+1}$  and  $s_{2m+1} - s_{2m+2} = a_{2m+2}$  so that  $|s_{2m+1} - s| \leq a_{2m+2}$ .

Thus in all cases  $|s_n - s| \leq a_{n+1}$ .

## Rocket Science 5

8.

$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\frac{d}{dx} \cosh x = \sinh x = \frac{e^x - e^{-x}}{2} \quad \begin{cases} > 0 & \text{if } x > 0 \\ = 0 & \text{if } x = 0 \\ < 0 & \text{if } x < 0 \end{cases}$$

Since  $\lim_{x \rightarrow \pm\infty} \cosh x = +\infty$ , the function  $\cosh x$  has an absolute minimum at  $x = 0$  and this minimum is  $\cosh 0 = 1$ .

## Section 14.2

4.  $(-1/2, 1/2)$

14.  $(-1, 1)$

16.  $(-2, 2]$ , conditional conv. at  $x = 2$

18.  $[-1, 1)$ , conditional conv. at  $x = -1$

24.  $1 \leq x \leq 5$

## Section 14.3

2.

$$(1 - x)^{-1} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$

$$\frac{d}{dx} (1 - x)^{-1} = (1 - x)^{-2} = \sum_{n=0}^{\infty} n x^{n-1}, \quad |x| < 1$$

$$\frac{d^2}{dx^2}(1-x)^{-1} = 2(1-x)^{-3} = \sum_{n=0}^{\infty} n(n-1)x^{n-2}, \quad |x| < 1$$

$$\frac{d^3}{dx^3}(1-x)^{-1} = 6(1-x)^{-4} = \sum_{n=0}^{\infty} n(n-1)(n-2)x^{n-3}, \quad |x| < 1$$

Therefore

$$(1-x)^{-4} = \frac{1}{6} \sum_{n=0}^{\infty} n(n-1)(n-2)x^{n-3} = \frac{1}{6} \sum_{m=0}^{\infty} (m+1)(m+2)(m+3)x^m, \quad |x| < 1,$$

where  $m = n - 3$

4.

$$(1-t)^{-1} = \sum_{n=0}^{\infty} t^n, \quad |t| < 1$$

Therefore,

$$\ln(1-t) = - \int_0^t \frac{dt}{1-t} = - \sum_{n=0}^{\infty} t^n dt = - \sum_{n=0}^{\infty} \frac{t^{n+1}}{n+1}, \quad |t| < 1$$

Thus

$$-\frac{\ln(1-t)}{t} = \sum_{n=0}^{\infty} \frac{t^n}{n+1}, \quad |t| < 1$$

and

$$-\int_0^x \frac{\ln(1-t)}{t} dt = \sum_{n=0}^{\infty} \int_0^x \frac{t^n}{n+1} dt = \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)^2} = \sum_{m=1}^{\infty} \frac{x^m}{m^2}, \quad |x| < 1$$

## Section 14.4

4a.

$$\sum_{n=0}^{\infty} \frac{x^{n+2}}{n!}$$

4b.

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^{n+1}}{n!}$$

**4c.**

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$$

**4d.**

$$\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1} x^{2n+2}}{(2n+1)!}$$

**4e.**

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$$

**4f.**

$$1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1} x^{2n}}{(2n)!}$$

**4g.**

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}$$

**4i.**

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

**6.**

$$\sec^2 x = \frac{d}{dx} \tan x = 1 + x^2 = \frac{2}{3} x^4 + \dots$$

**14.**

$$\begin{aligned} \arcsin x &= \int_0^x \frac{dx}{(1-x^2)^{1/2}} = \int_0^x \left( \sum_{n=0}^{\infty} \frac{(-1)^n [1 \cdot 3 \cdot 5 \cdots (2n-1)] x^n}{2^n (n)!} \right) dx \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n [1 \cdot 3 \cdot 5 \cdots (2n-1)] x^{n+1}}{2^n (n+1)!} \end{aligned}$$

## Section 14.5

- 2.** .36788,  $n = 7$
- 4.** .0523359, 3 terms
- 10.**  $|R_3| \approx 1.0 \times 10^{-6}$
- 12.**  $x = 1/9$ ,  $\ln 1.25 \approx .22314$  in both cases

### Section 16.1

- 2c.**  $[4, \pi/6]$ ,  $[-4, 7\pi/6]$
- 2d.**  $[4, \pi/3]$ ,  $[-4, 4\pi/3]$

### Section 16.2

Sorry that I haven't time to put the graphs online.

- 2a.** single loop
- 4c.** single loop
- 4e.** single loop
- 4f.** double loop
- 4g.** 4 petal rose
- 4l.** heart shaped
- 4m.** curve goes to infinity along both sides of positive y axis and is asymptotic to the axis.
- 6a.**  $x^2 + y^2 = 4$
- 6b.**  $x = y$

**6c.**  $x = 3$

**6d.**  $x^2 + y^2 = 4y$

**6f.**  $(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$

**6g.**  $(x^2 + y^2)^3 = 4a^2x^2y^2$

**6h.**  $x^2(x^2 + y^2) = y^2$

**6i.**  $(x^2 + y^2)^3 = (x^2 - y^2)^2 - 4x^2y^2$

## Section 16.4

**8.** The curves intersect at the 4 points where

$$\theta = \pm\frac{\pi}{6}, \quad \pm\frac{5\pi}{6}.$$

The angle between the two tangent lines at each intersection point is

$$\frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}.$$

## Section 17.5

**2b.**  $r = \sqrt{2}$

## Section 17.6

6.

$$\mathbf{v} = (\cos t - t \sin t) \mathbf{i} + (\sin t + t \cos t) \mathbf{j}, \quad \mathbf{a} = (-2 \sin t - t \cos t) \mathbf{i} + (2 \cos t - t \sin t) \mathbf{j},$$

$$\frac{d^2 s}{dt^2} = \frac{t}{\sqrt{t^2 + 1}}, \quad \kappa ||\mathbf{v}||^2 = \frac{t^2 + 2}{\sqrt{t^2 + 1}}$$

### Section 18.6

2. right circular cone

### Section 18.7

2b.  $(x, y, z) = (-3/2, \sqrt{3}/2, 11)$

### Section 19.1

2.  $xy \neq 0$

4. all  $(x, y)$

8. All points except those on lines  $y = \pm 2x$

14. circular paraboloid

18. plane

### Section 19.2

6.

$$z_x = 3 \sec^2 3x, \quad z_y = -4 \csc^2 4y$$

10.

$$z_x = ye^{xy} + xy^2 e^{xy}, \quad z_y = xe^{xy} + x^2 y e^{xy}$$

16.

$$w_x = \frac{1}{\sqrt{1 - (\frac{z}{xy})^2}} \left( \frac{-z}{x^2 y} \right), \quad w_y = \frac{1}{\sqrt{1 - (\frac{z}{xy})^2}} \left( \frac{-z}{xy^2} \right), \quad w_z = \frac{1}{\sqrt{1 - (\frac{z}{xy})^2}} \left( \frac{1}{xy} \right)$$

### Section 19.3

2.

$$-x - 16y + z = -4$$

8.

$$-x + y + 8z = 2\pi$$

### Section 19.5

2a.  $D_{\hat{\mathbf{u}}} f(1, 1, 2) = 11/\sqrt{6}$

2c.  $D_{\hat{\mathbf{u}}} f(1, 0, 0) = 1/2$

2d.  $D_{\hat{\mathbf{u}}} f(1, 0, 0) = 2/\sqrt{6}$

8. tangent plane;  $4x + 5y + 6z = 77$

normal line:  $x = 4 + 8t, \quad y = 5 + 10t, \quad z = 6 + 12t$

### Section 19.7

18.  $(s, t) = (1/2, 1/2), D_{\min} = 2\sqrt{3}$

22.

$$(x, y) = \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$