Name: ______________________________

Section: ______________________________

Math 1572H. Practice Midterm Exam I

There are a total of 100 points on this exam, plus one 5-point extra credit problem that you should only work if you complete the rest of the exam. To get full credit for a problem you must show the details of your work. Answers unsupported by by an argument will get little credit. The last page of the exam contains results from the course that you may, or may not, want to use.

Problem	Score
1.	______
2.	______
3.	______
4.	______
5.	______
6.	______
Total:	______

Problem 1 *(20 points) Use the substitution* $x^2 = \tan\theta$ *to evaluate the integral*

$$\int \frac{\sqrt{x^4+1}}{x^3}\,dx.$$

Show all steps.

Problem 2 *Recall that the maximal error bound for the trapezoidal rule is*

$$|\int_a^1 f(x)\ dx - T_n| = |E_T| \leq \frac{K(b-a)^3}{12n^2},$$

where $K = \max_{x\in[a,b]} |f''(x)|$.

a. (10 points) *Let* $f(x) = 18 + 10x + 3x^2 - 2x^3$. *Find the maximum value* K *of* $|f''(x)|$ *on the interval* $0 \leq x \leq 2$.

b. (10 points) *Use the result of part a. to find an error bound for* $|E_T|$ *corresponding to the integral*

$$\int_0^2 \left(18 + 10x + 3x^2 - 2x^3\right)\ dx$$

when $n = 150$.

Problem 3 *Find each of the following limits.*

a. (10 points)

$$\lim_{x\to 0} \frac{\tan 2x - 2x}{x^3}$$

b. (10 points)

$$\lim_{x\to 0} (e^x + x)^{1/x}$$

Problem 4 *The region bounded by the cubic* $y = x^3 - 3x$ *and the line* $y = x$ *(and on the right of the* y*-axis) is covered by a lamina of constant density* ρ*.*

a. (15 points) *Find* M_x, M_y *and the mass* M *for this lamina.*

b. (5 points) *Compute the center of mass* $(\bar{x}, \bar{y})$ *of the lamina.*

Problem 5 *(20 points) Determine if the improper integral*

$$\int_0^\infty \frac{1}{(1+x)(1+x^2)}\ dx$$

converges and, if so, give the steps in its evaluation.

Problem 6 *(EXTRA CREDIT: 5 points) Given* $f(1) = 3$, $f(5) = 10$, $f'(1) = 4$, $f'(5) = 7$, $\int_1^5 f(x)\ dx = 15$, $\int_1^5 f'(x)\ dx = 7$, $\int_1^5 x^2 f(x)\ dx = 25$, *find the value of*

$$\int_1^5 x f'(x)\ dx.$$

Some formulas that you may, or may not, want to use are

$$\int u\ dv = uv - \int v\ du$$

$$\sin 2x = 2\sin x\cos x, \quad \cos 2x = \cos^2 x - \sin^2 x,$$

$$2\sin^2 x = 1 - \cos 2x, \quad 2\cos^2 x = 1 + \cos 2x,$$

$$M_x = \frac{1}{2}\int \rho y^2\ dx,\ M_y = \int \rho xy\ dx,\ M = \int \rho y\ dx, \bar{x} = \frac{M_y}{M},\ \bar{y} = \frac{M_x}{M}$$

$$T_n = \frac{\Delta x}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n), \quad \Delta x = \frac{b-a}{n}$$

$$S_n = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n), \quad \Delta x = \frac{b-a}{n}, n \text{ even}$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| + c, \quad \int \csc x\ dx = \ln|\csc x - \cot x| + c,$$

Brief solutions:

1.
$$\frac{1}{2}\ln\left|x^2+\sqrt{x^4+1}\right|-\frac{\sqrt{x^4+1}}{2x^2}+C$$

2. a. $K=18$. b. $|E_Y|\leq\frac{1}{1875}\approx 5.333\times 10^{-4}$

3. a. $8/3$ b. e^2

4. a.
$$M_x=-\frac{64}{105}\rho,\quad M_y=\frac{64}{15}\rho,\quad M=4\rho$$

   b.
$$(\bar{x},\bar{y})=(\frac{16}{15},-\frac{16}{105})$$

5. $\pi/4$

6. 32