Name: ______________________________

Section: ______________________________

Math 1572H. Practice Midterm Exam II

There are a total of 100 points on this exam, plus one 5-point extra credit problem that you should only work if you complete the rest of the exam. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit. The last page of the exam contains results from the course that you may, or may not, want to use.

Problem	Score
1.	
2.	
3.	
4.	
5.	
6.	
7.	
Total:	

Note: This practice exam contains the same number of points but one more problem than the actual exam. I did this to give you practice in the total variety of problems that might occur on the midterm and the final.

Problem 1 *(15 points) Determine if the following alternating series is convergent.*

$$\sum_{n=1}^{\infty} (-1)^n \frac{8n+5}{n(2n+1)}.$$

Justify your answer.

Problem 2 *(20 points) Consider the series*

$$\sum_{n=0}^{\infty} \frac{(n+1)x^n}{3^n(3n+1)}.$$

a. *Find the radius of convergence R for this series.*

b. *What is the precise interval of convergence (including an examination of the endpoints)?*

Problem 3 *(20 points) Determine if each of the following infinite series converges or diverges. In each case give a reason for your answer:*

a.

$$\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$$

b.

$$\sum_{n=1}^{\infty} \frac{1}{\ln(3n^2)}$$

c.

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$$

Problem 4 *(20 points) If* $|x| \leq 1$ *find the smallest value of* N *such that you can guarantee*

$$\left| \sin x - \sum_{k=0}^{N} (-1)^k \frac{x^{2k+1}}{(2k+1)!} \right| < 10^{-10}.$$

Note that this is an alternating series. Show your work.

Problem 5 *(10 points) The Taylor polynomial $T_5(x)$ about $x = 0$ for the function $f(x) = 1/(4+x)$ is*

$$\frac{1}{4+x} \approx T_5(x) = \frac{1}{4} - \frac{x}{4^2} + \frac{x^2}{4^3} - \frac{x^3}{4^4} + \frac{x^4}{4^5} - \frac{x^5}{4^6}.$$

Use this polynomial to obtain the Taylor polynomial $T_4(x)$ for $1/(4+x)^2$.

Problem 6 *(10 points) Write the repeating decimal $R = .123123123\cdots$ as an infinite geometric series with common ratio $r = 1/1000$. Then sum the series to find the rational number R.*

Problem 7 *EXTRA CREDIT (5 points) Consider the infinite series*

$$4 + 5x + 6x^2 + 7x^3 + \cdots = \sum_{n=0}^{\infty} (4+n)x^n.$$

Find the sum of the series in the interval where it converges.

Some results that you may, or may not, want to use are

$$\int u\, dv = uv - \int v\, du, \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1,$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \ |x| < 1, \quad \exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!},$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}, \quad |x| < 1$$

$$f(x) = \sum_{n=0}^{N} \frac{f^{(n)}(0)}{n!} x^n + R_N(x),$$

$$\text{where } R_N(x) = \int_0^x \frac{f^{(N+1)}(t)}{N!}(x-t)^N\, dt = f^{(N+1)}(\xi)\frac{x^{N+1}}{(N+1)!}$$

p-series: $\sum_{n=1}^{\infty} 1/n^p$ diverges for $0 < p \leq 1$ and converges for $p > 1$.

The alternating series $S = \sum_{n=1}^{\infty}(-1)^n a_n$ converges if 1) $a_n \geq 0$ for all n, 2) $a_{n+1} \leq a_n$ for all n, and 3) $\lim_{n\to\infty} a_n = 0$. The error $E_N = S - \sum_{n=0}^{N}(-1)^n a_n$ has the bound $|E_N| \leq a_{N+1}$

Brief answers:

1. convergent
2. a. $R = 3$, b. $(-3, 3)$
3. a. diverges, b. diverges, c. converges
4. $N = 6$
5. $$T_4(x) = \frac{1}{4^2} - \frac{2}{4^3}x + \frac{3}{4^4}x^2 - \frac{4}{4^5}x^3 + \frac{5}{4^6}x^4$$
6. $41/333$
7. $$\frac{4-3x}{(1-x)^2}, \quad -1 < x < 1$$