Name: _____

Section: _____

Math 2243. Lecture 020 Practice Midterm I

(There are a total of 100 points on this exam)

It is important that you show your work on each problem. Answers unsupported by details will receive little credit.

Problem 1 *Consider the differential equation and initial condition*

$$\frac{dy}{dt} = \frac{y}{t}, \qquad y(0) = 0.$$

a. (10 points) *Do the assumptions of the Picard Theorem hold for this equation and initial condition?*

b. (15 points) *Find the general solution of the equation and determine which solutions, if any, satisfy* $y(0) = 0$.

Problem 2 *(25 points) Find the general solution of the equation*

$$\frac{dy}{dt} = \frac{t}{y + ty}.$$

Problem 3 *(25 points) Find the general solution of the linear differential equation*

$$y' + \frac{t}{t^2 + 1} y = t.$$

Problem 4 *(25 points) Suppose $P(t)$ represents the principal of a loan $P_0 = \$100,000$ after t years. Interest is charged at a rate $r = .05$ (5% per year) and payments are made at the constant and continuous rate M (dollars per year). Then $P(t)$ obeys the differential equation*

$$\frac{dP}{dt}(t) = rP(t) - M$$

with initial condition $P(0) = P_0$. What value should M have if the loan is to be paid in full at the end of $T = 20$ years – that is $P(T) = 0$?

SOLUTIONS:

1. Picard's theorem doesn't apply, because $f(t, y) = y/t$ isn't defined at $(0, 0)$. Thus the theorem doesn't guarantee either existence or uniqueness. However, the general solution is $y = ct$ for c an arbitrary constant, so there are an infinite number of solutions y such that $y(0) = 0$.

2.
$$y(t)^2 = 2t - \ln(1+t)^2 + c.$$

3.
$$y = \frac{1}{3}(t^2+1) + \frac{c}{\sqrt{t^2+1}}.$$

4.
$$M = \frac{P_0 r e^{rT}}{e^{rT} - 1}.$$

or
$$M = \frac{(100000)(.05)e}{e-1} \sim \$7,909.88.$$