Name: ______________________________

Section: ______________________________

Math 2243. Lecture 020   Practice Midterm Exam II

There are a total of 100 points on this exam. The first problem is multiple choice, and you should circle the correct answer. There will be no partial credit for this problem. There will be partial credit awarded for the remaining problems but you must show your work.

Problem 1 (15 points) *A population of bacteria $y(t)$ changes according to the logistics equation*

$$y' = 100y(1 - \frac{y}{200}).$$

Time t is measured in days and y is measured in units of a thousand. If the intial population is 300, after 6 months the population is approximately

1. *200 (thousand)*
2. *100 (thousand)*
3. *50 (thousand)*
4. *500 (thousand)*

Problem 2

$$A = \begin{pmatrix} 5 & 6 \\ -1 & 11 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

Compute each of the following, if it exists.

a. (6 points) AB

b. (6 points) $\det(CB)$

c. (6 points) $3A - 2B$

d. (7 points) $C^{\mathrm{T}}B$

Problem 3 (20 points) *Use Gauss-Jordan elimination to find the general solution of the system*

$$\begin{array}{ccccccc} x & - & y & + & z & = & 1 \\ x & + & y & & & = & 2 \\ x & + & 2y & - & z & = & 5 \end{array}$$

Problem 4 (20 points) *Let $V = P_2$ be the space of all polynomials $p(t)$ of order ≤ 2. Is the set*

$$\{-t+1, \quad t^2+1, \quad 2t^2+2t-1\}$$

linearly independent? Does it form a basis for P_2? Justify your answer.

Problem 5 (20 points) *Find a basis for the solution space of the system of equations*

$$\begin{array}{rcrcrcl} x & - & y & + & z & = & 0 \\ x & + & y & & & = & 0 \\ 3x & + & y & + & z & = & 0 \\ & & 2y & - & z & = & 0 \end{array}$$

Solutions:

1. 1

2. **a.**

$$\begin{pmatrix} 10 & 6 \\ -2 & 11 \end{pmatrix}$$

b.

Doesn′t exist.

c.

$$\begin{pmatrix} 11 & 18 \\ -3 & 31 \end{pmatrix}$$

d.

$$\begin{pmatrix} 2 & 6 \end{pmatrix}$$

3.

$$x = 4, \quad y = -2, \quad z = -5$$

4. Linearly independent. Forms a basis since P_2 is 3-dimensional.

5. Solution space is

$$(x, y, z) = c(-\frac{1}{2}, \frac{1}{2}, 1), \quad c \text{ arbitrary}$$

so a basis is a single vector

$$\{(-\frac{1}{2}, \frac{1}{2}, 1)\}.$$