Name: ______________________________

Math 4242/4457 Sec. 10 Practice Midterm Exam I

There are a total of 100 points on this 55 minute exam. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit. A standard calculator and ONE 8.5×11 inch sheet of notes are allowed, but no books, other notes, cell phones or other elecronic devices are allowed. Do all of your calculations on this test paper.

Problem	Score
1.	________
2.	________
3.	________
4.	________
5.	________
Total:	________

Problem 1 *Let*

$$A = \begin{pmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{pmatrix}.$$

a. (15 points) *Use the Gauss-Jordan method to compute A^{-1}.*

b. (5 points) *Solve the linear system*

$$\begin{aligned} 3x_1 - x_2 + 4x_3 &= 1 \\ 2x_2 + x_3 &= 2 \\ x_1 - x_2 - 2x_3 &= -1. \end{aligned}$$

by using the inverse of the coefficient matrix.

Problem 2

$$A = \begin{pmatrix} -2 & 1 \\ 2 & 5 \end{pmatrix}.$$

a. (10 points) *Compute the LU factorization of A.*

b. (10 points) *Use the LU factorization of A to solve the equation $Ax = b$, where*

$$b = \begin{pmatrix} 5 \\ 1 \end{pmatrix}.$$

Follow the steps $Lc = b,\ Ux = c$.

Problem 3 *(a) (10 points) Draw the digraph represented by the incidence matrix*

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{pmatrix},$$

labeling the vertices and directed edges.

(b) (10 points) Determine the rank of A and dim coker(A), with a minimum of computation.

Problem 4 *(20 points) For which values of a is the matrix*

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

positive definite?

Problem 5

$$A = \begin{pmatrix} 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 1 & 0 & 0 & -1 \end{pmatrix}.$$

a.(5 points) *Determine the rank of* A*.*

b. (5 points) *Determine the dimension of coker* (A)*.*

c. (5 points) *Find a basis for ker* (A)*.*

d. (5 points) *Find the general solution of the equation*

$$Ax = b, \qquad b = \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}.$$

Brief solutions:

1.

$$A^{-1} = \begin{pmatrix} 1/6 & 1/3 & 1/2 \\ -1/18 & 5/9 & 1/6 \\ 1/9 & -1/9 & -1/3 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 1/3 \\ 8/9 \\ 2/9 \end{pmatrix}.$$

2.

$$L = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} -2 & 1 \\ 0 & 6 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

3. m=5 edges, n=5 vertices, 1 loop. dim ker$(A) = 1$, r=rank (A)=4, dim coker $(A) = m - r = 1$.

4.
$$-1 < a < 1.$$

5. a): rank $=r = 3$. b): $m = 3$, $n = 5$, dim coker $(A) = m - r = 0$.
c): basis
$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 13 \\ -6 \\ 1 \end{pmatrix}.$$

d):
$$\mathbf{x} = \begin{pmatrix} 0 \\ 3 \\ 12 \\ -5 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 13 \\ -6 \\ 1 \end{pmatrix}.$$