

Homework Problem Set #4

Math 5467

March 20, 2006

Exercise 1 Find the z -transform for the sequence $\mathbf{x}(n) = 0$ for $n < 0$ and $\mathbf{x}(n) = (2/3)^n$ for $n \geq 0$, i.e.,

$$\mathbf{x} = \left(\cdots \ 0 \ 0 \ 1 \ \frac{2}{3} \ \frac{4}{9} \ \cdots \ \frac{2^n}{3^n} \ \cdots \right)$$

Exercise 2 Show explicitly that the double-shift orthogonality condition in the frequency domain

$$\int_{-\pi}^{\pi} e^{2ik\omega} |C(\omega)|^2 d\omega = 2\pi \delta_{k0}$$

for integer k , is equivalent to

$$|C(\omega)|^2 + |C(\omega + \pi)|^2 = 2.$$

Exercise 3 Plot the frequency response function $|H(\omega)|$ in the interval $0 \leq \omega \leq 2\pi$ for the Daubechies 4 tap filter. Point out the features of the graph that cause this filter to belong to the maxflat class.

Exercise 4 Let $\phi(t)$ and $w(t)$ be the Haar scaling and wavelet functions. Let V_j and W_j be the spaces generated by $\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k)$ and $w_{j,k}(t) = 2^{j/2}w(2^j t - k)$, $k = 0, \pm 1, \dots$, respectively. Let $f(t)$ be defined on $0 \leq t < 1$ and given by

$$f(t) = \begin{cases} 4 & 0 \leq t < 1/4 \\ 1 & 1/4 \leq t < 1/2 \\ -1 & 1/2 \leq t < 3/4 \\ 2 & 3/4 \leq t < 1. \end{cases}$$

1. Express f in terms of the basis for V_2 .
2. Decompose f into its component parts in W_1 , W_0 , and V_0 . In other words, find the Haar wavelet decomposition for f .
3. Sketch each of the four decompositions.

Exercise 5 Let $\phi(t)$ and w be the Haar scaling and wavelet functions, respectively. Let V_j and W_j be the spaces generated by $\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k)$ and $w_{j,k}(t) = 2^{j/2}w(2^j t - k)$, $k = 0, \pm 1, \dots$, respectively. Suppose the real valued function $f(t) = \sum_k a_k \phi_{1,k}(t)$ belongs to V_1 and $f \perp V_0$. Show explicitly that $a_{2\ell+1} = -a_{2\ell}$ for all integers ℓ . Conclude that f can be expressed as a linear combination of the functions, $w_{0,k}(t)$, hence that $f \in W_0$.