6983 Special Problems

1. Find the general solution of the following differential equation:

$$x\frac{dy}{dx} - 2y = 12x^3 \cos 3x.$$

2. Solve the initial value problem:

$$x\frac{dy}{dx} + 3y = 8x \text{ and } y(1) = 6.$$

3. Solve the initial value problem:

$$\frac{dy}{dt} = 12 - \frac{6y}{(100 + 2t)} \text{ and } y(0) = 100.$$

4. A tank contains 25 kg of salt dissolved in 4,000 liters of water. Brine that contains (1/10) kg of salt per liter of water enters the tank at the rate of 20 liters/min. Brine is drained from the tank at the slower rate of 15 liters/min. Find an expression for the amount of salt in the tank at time t.

5. A tank contains 80 lbs of salt dissolved in 600 gallons of water. Brine that contains (1/3) lbs of salt per gallon of water enters the tank at the rate of 15 gal/min. Brine is drained from the tank at the slower rate of 10 gal/min. Find an expression for the amount of salt in the tank at time t.

6. The following is the value chart for a certain polar function $r = f(\theta)$. Sketch the graph of this function.

θ	r
0	2
$\pi/4$	3.4
$\pi/2$	4
$3\pi/4$	3.4
π	2
$5\pi/4$	0.6
$3\pi/2$	0
$7\pi/4$	0.6
2π	2

7. The following is the value chart for a certain polar function $r = f(\theta)$. Sketch the graph of this function.

θ	r
0	3
$\pi/6$	2.6
$\pi/3$	1.5
$\pi/2$	0
$2\pi/3$	-1.5
$5\pi/6$	-2.6
π	-3
$7\pi/6$	-2.6
$4\pi/3$	-1.5
$3\pi/2$	0
$5\pi/3$	1.5
$11\pi/6$	2.6
2π	3

8. Show that the following vector equations are each a vector equation for the same line.

$$\vec{r}(t) = (-2t+3)\vec{i} + (2t-5)\vec{j}$$
$$\vec{R}(s) = (2s+1)\vec{i} + (-2s-3)\vec{j}$$

9. Show that the following vector equations are each a vector equation for the same parabola. Recall that a parabola is determined by three points.

$$\vec{r}(t) = (t^2-3t)\vec{i} + (4-t^2)\vec{j}$$
$$\vec{R}(s) = (s^2-7s+10)\vec{i} + (4s-s^2)\vec{j}$$

10. Consider the parabola which is the graph of the vector equation

$$\vec{r}(t) = (t^2+3t-4)\vec{i} + (6+4t-t^2)\vec{j}.$$

Find the vector equation of the tangent line to this curve at the point (6,10).

11. Consider the curve which is the graph of the vector equation

$$\vec{r}(t) = (t^3-t^2+4)\vec{i} + (-t^2+3t+3)\vec{j}.$$

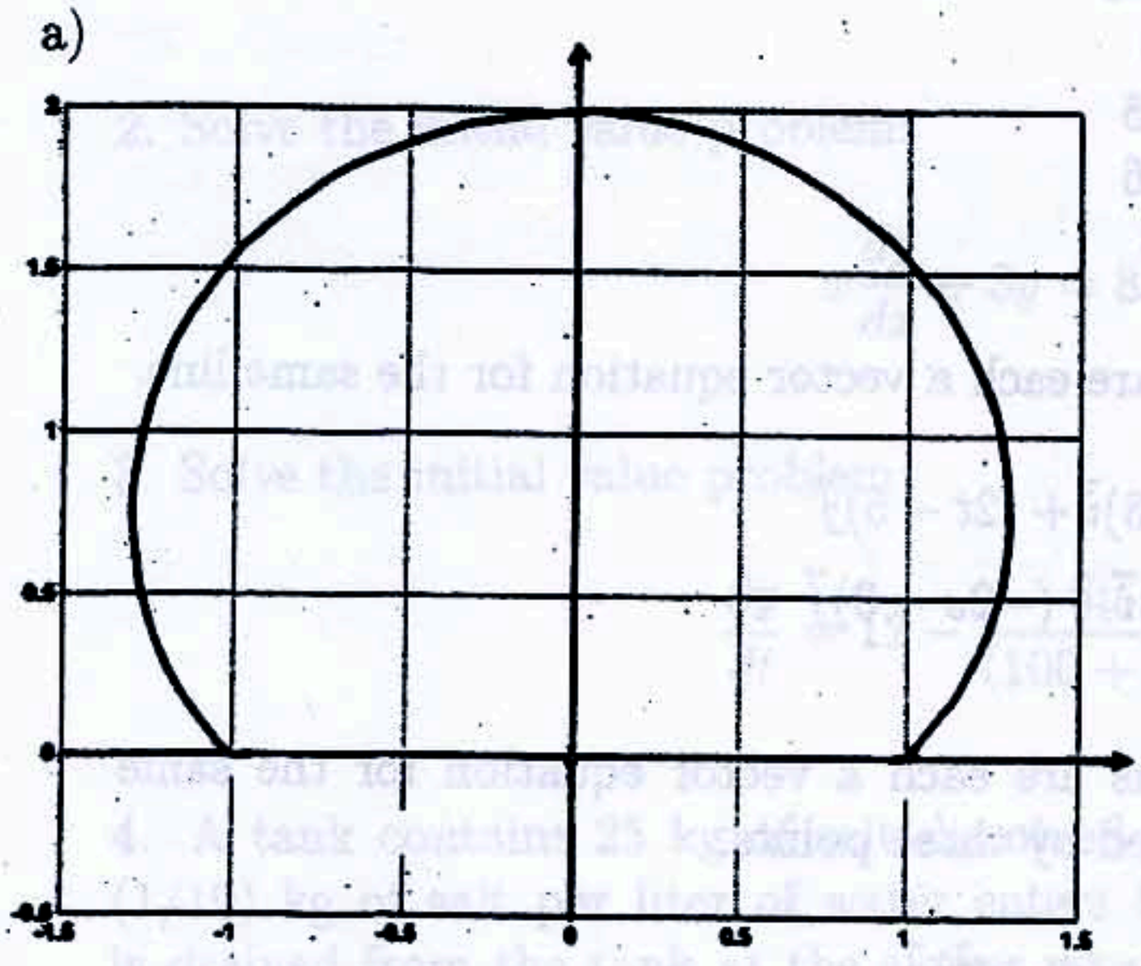
Find the vector equation of the tangent line to this curve at the point corresponding to $t = 2$.

12. Find four different sets of polar coordinates for the following points. All coordinates must be such that $-2\pi < \theta \leq 2\pi$. Give exact values of θ in terms of π.

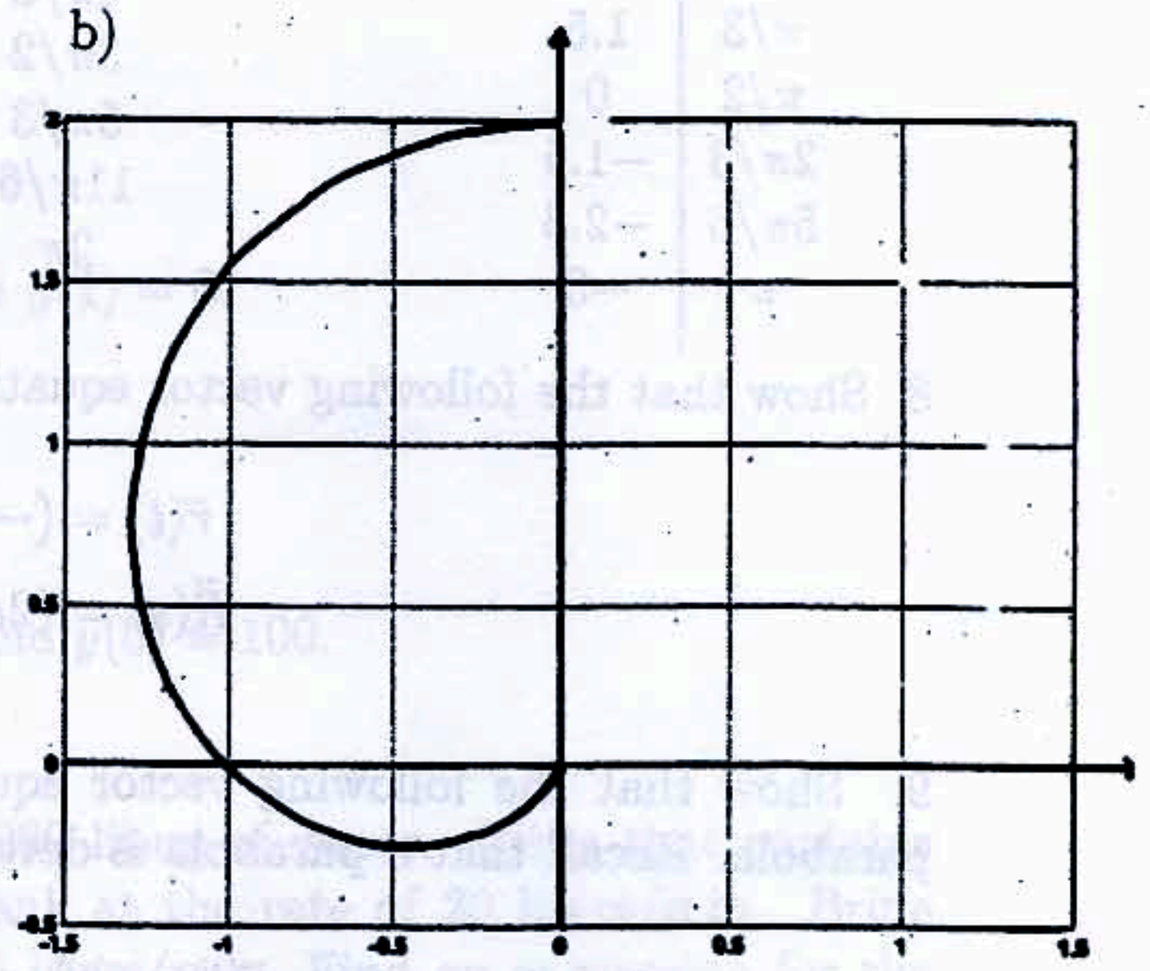
a) $(-\sqrt{2}, \sqrt{2})$ b) $(\sqrt{3}, -1)$

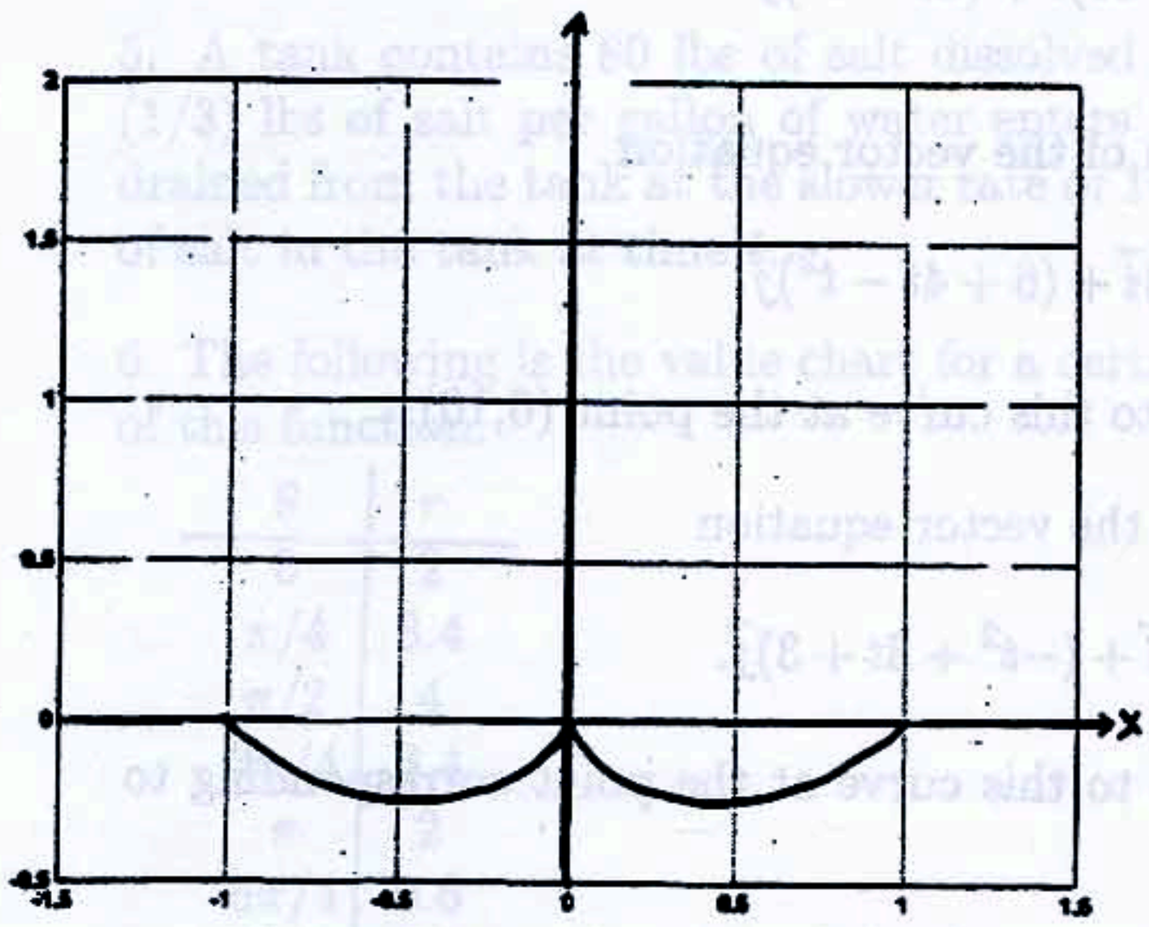
13. 25. Below are some parts of the graph of the polar equation $r = 1 + \sin\theta$. In each case state exactly the values of θ which correspond to the section of the graph shown.

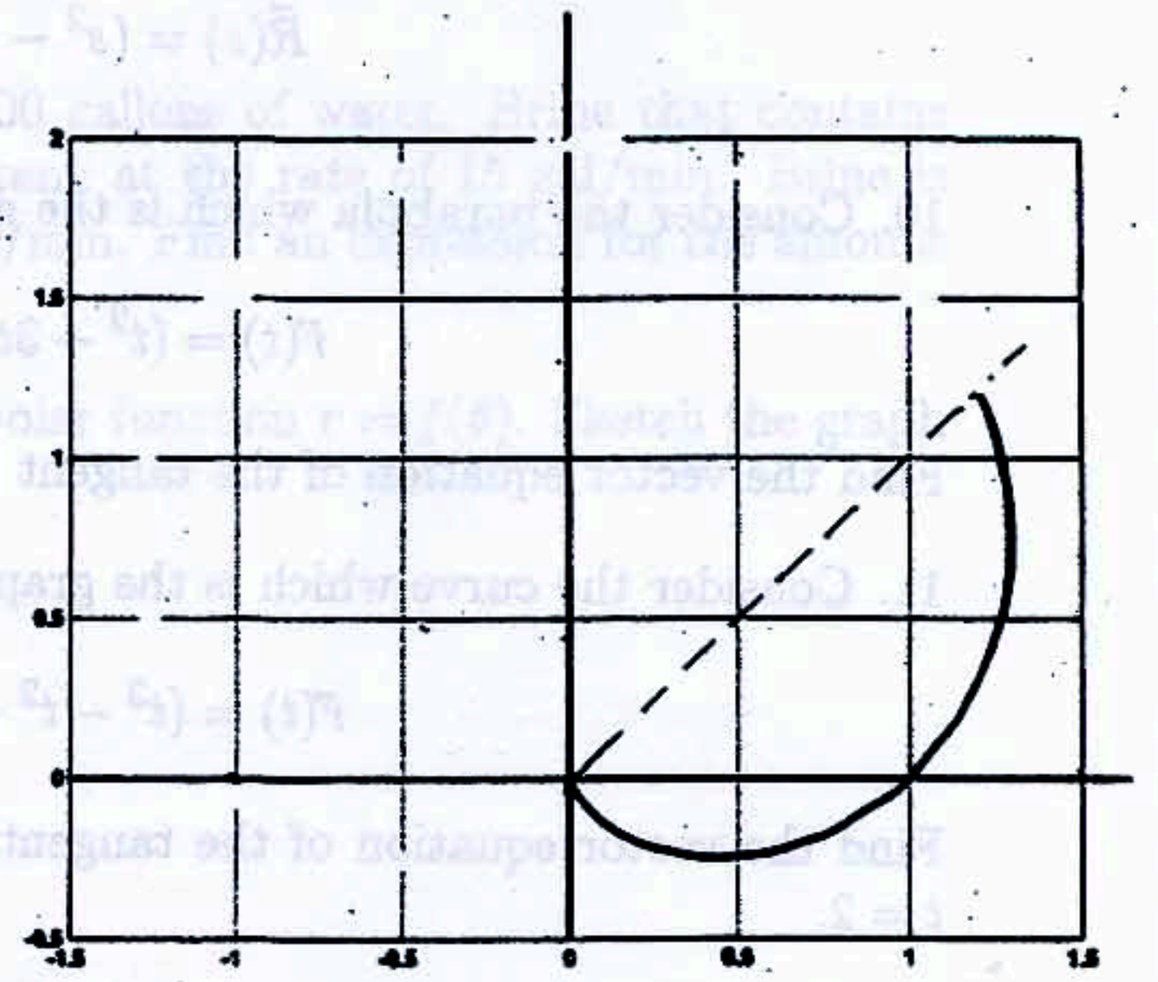
a)



b)

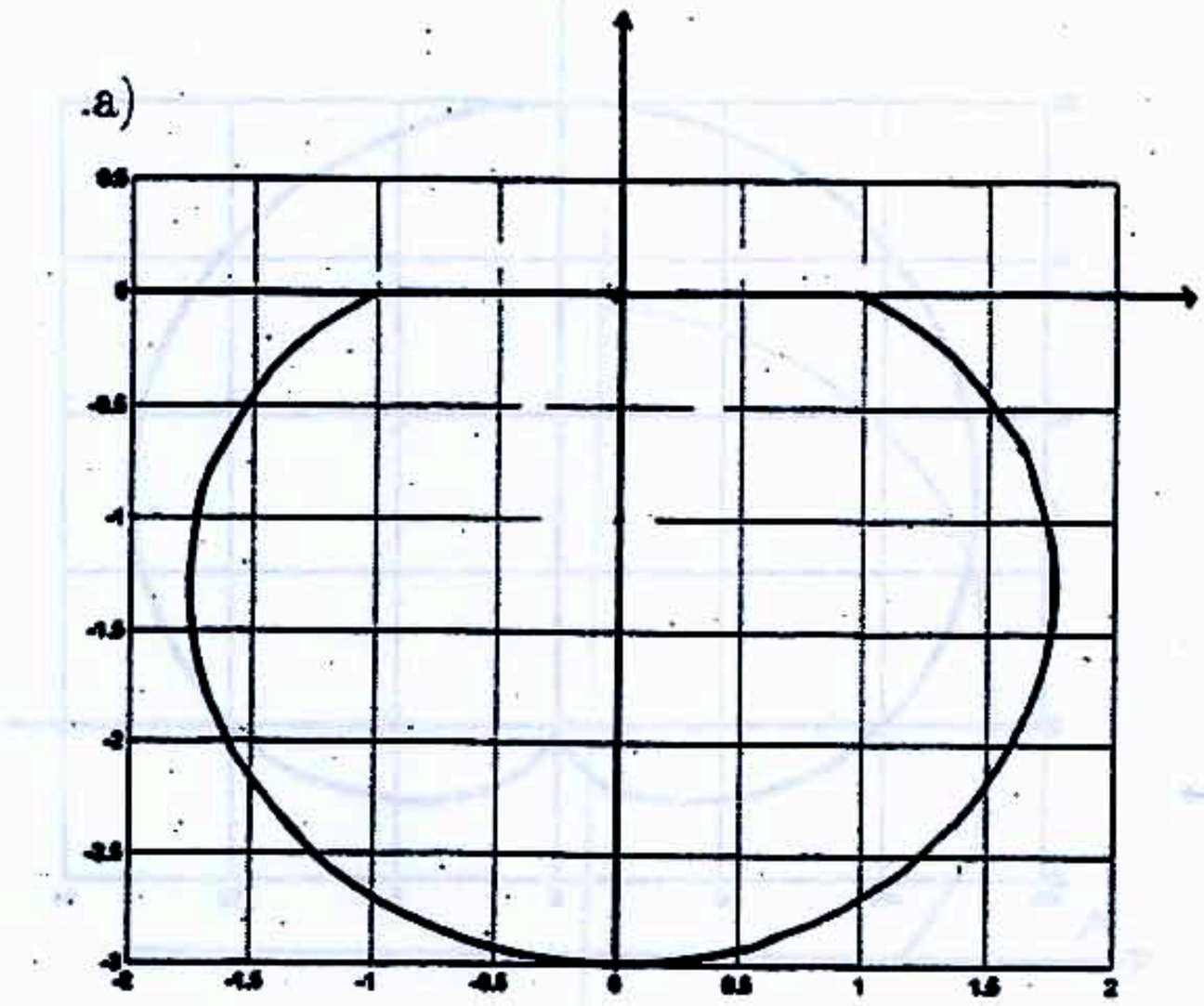




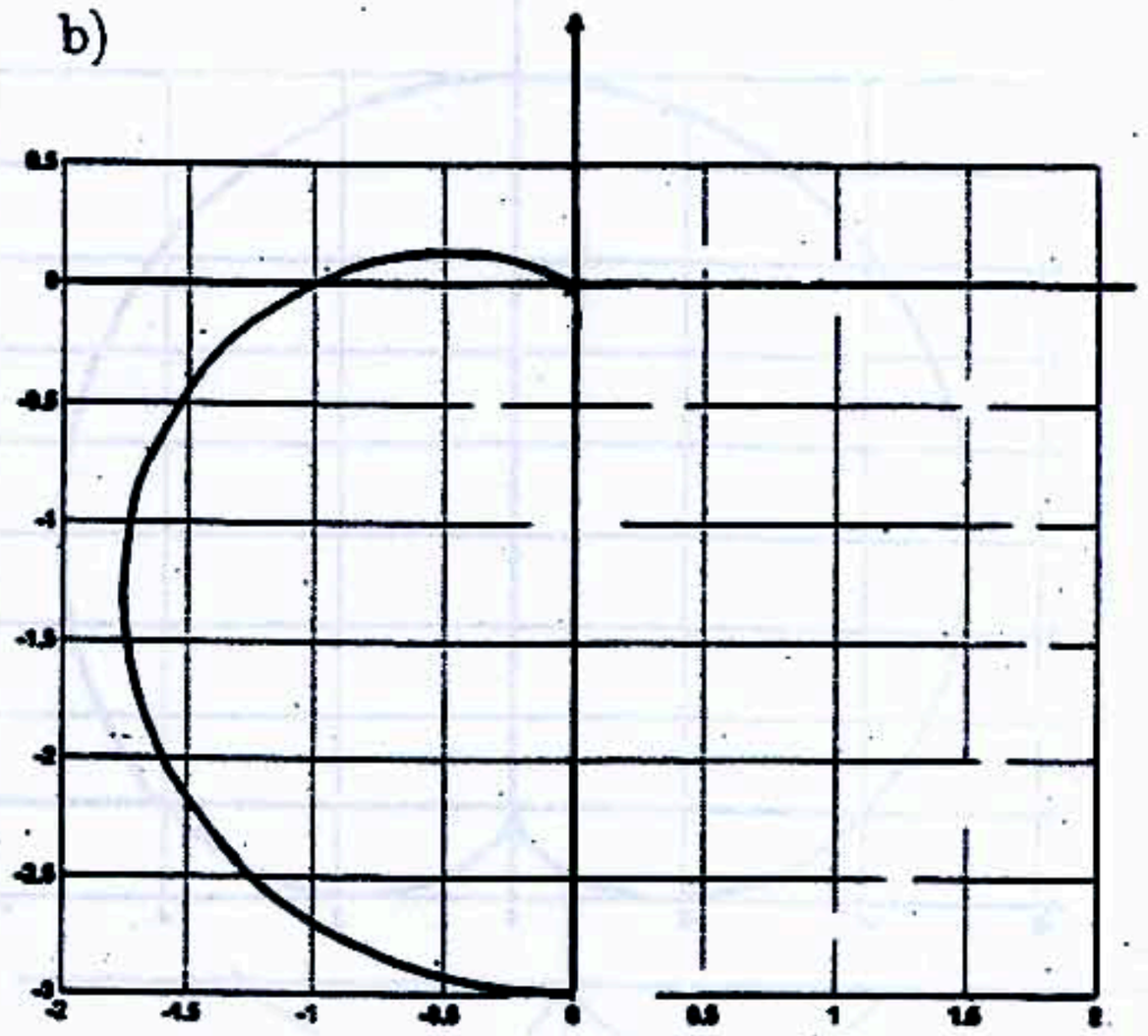


14. Below are some parts of the graph of the polar equation $r = 1 - 2\sin\theta$. State exactly the values of θ which correspond to the sections of the graph shown.

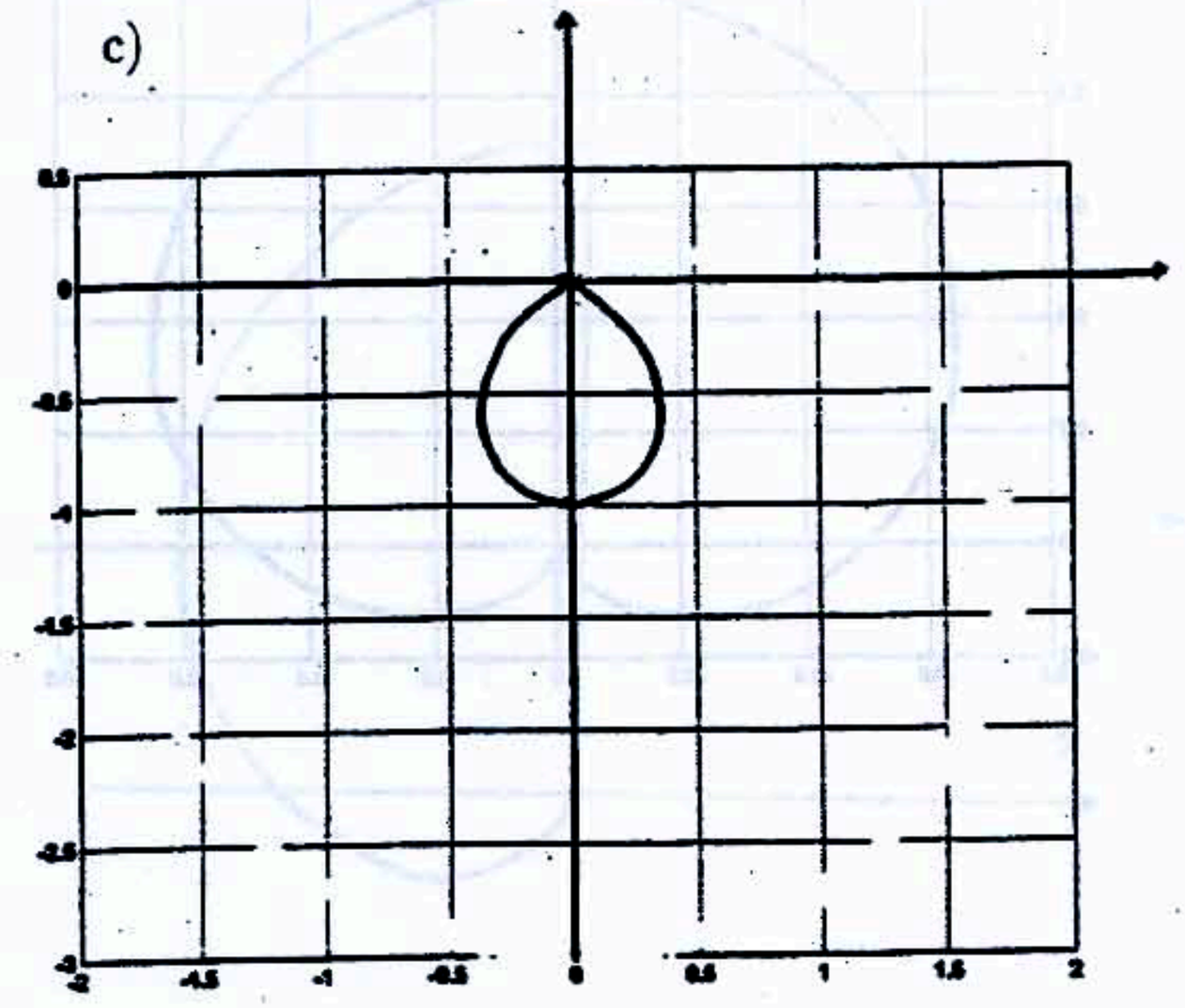
a)



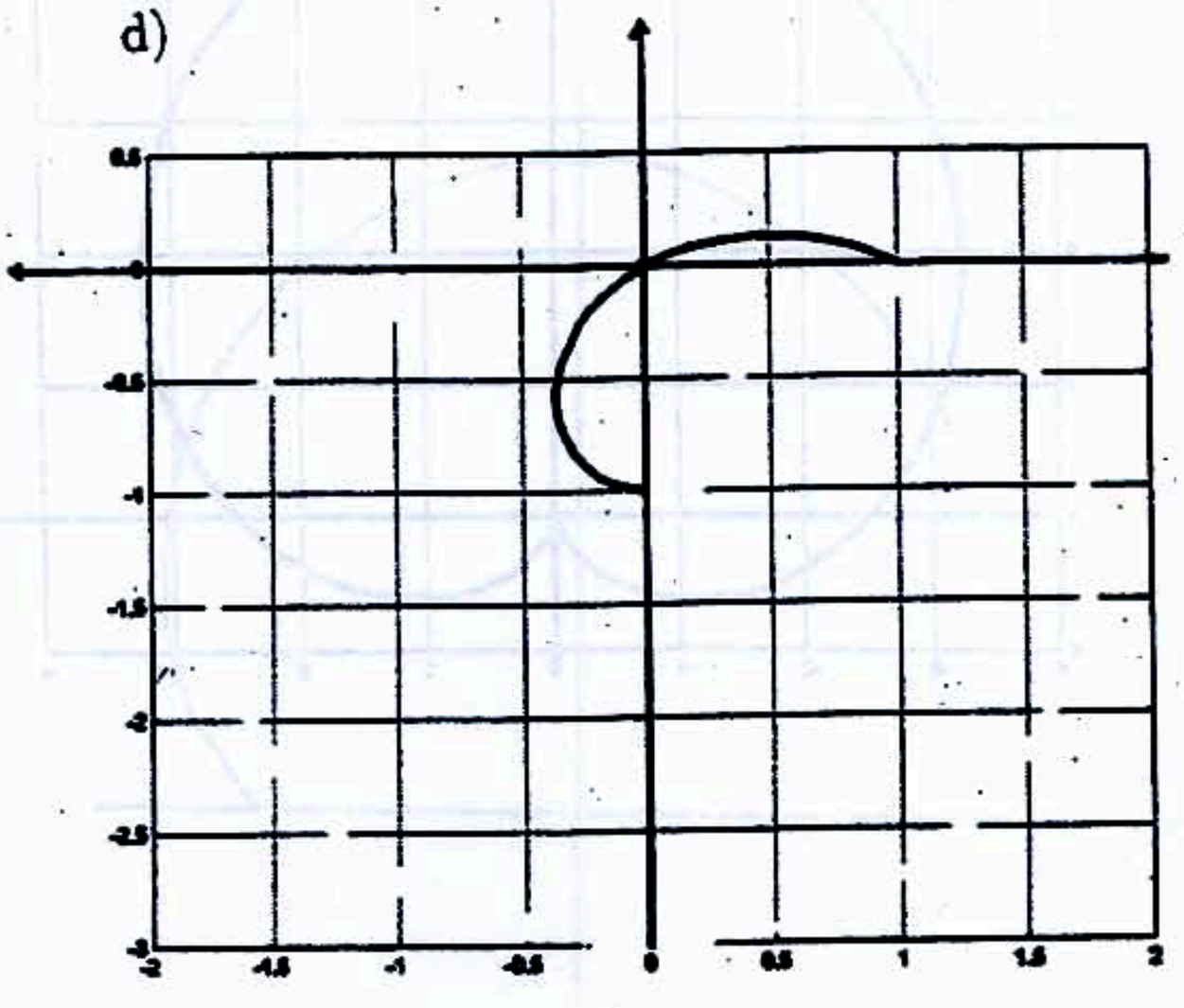
b)



c)

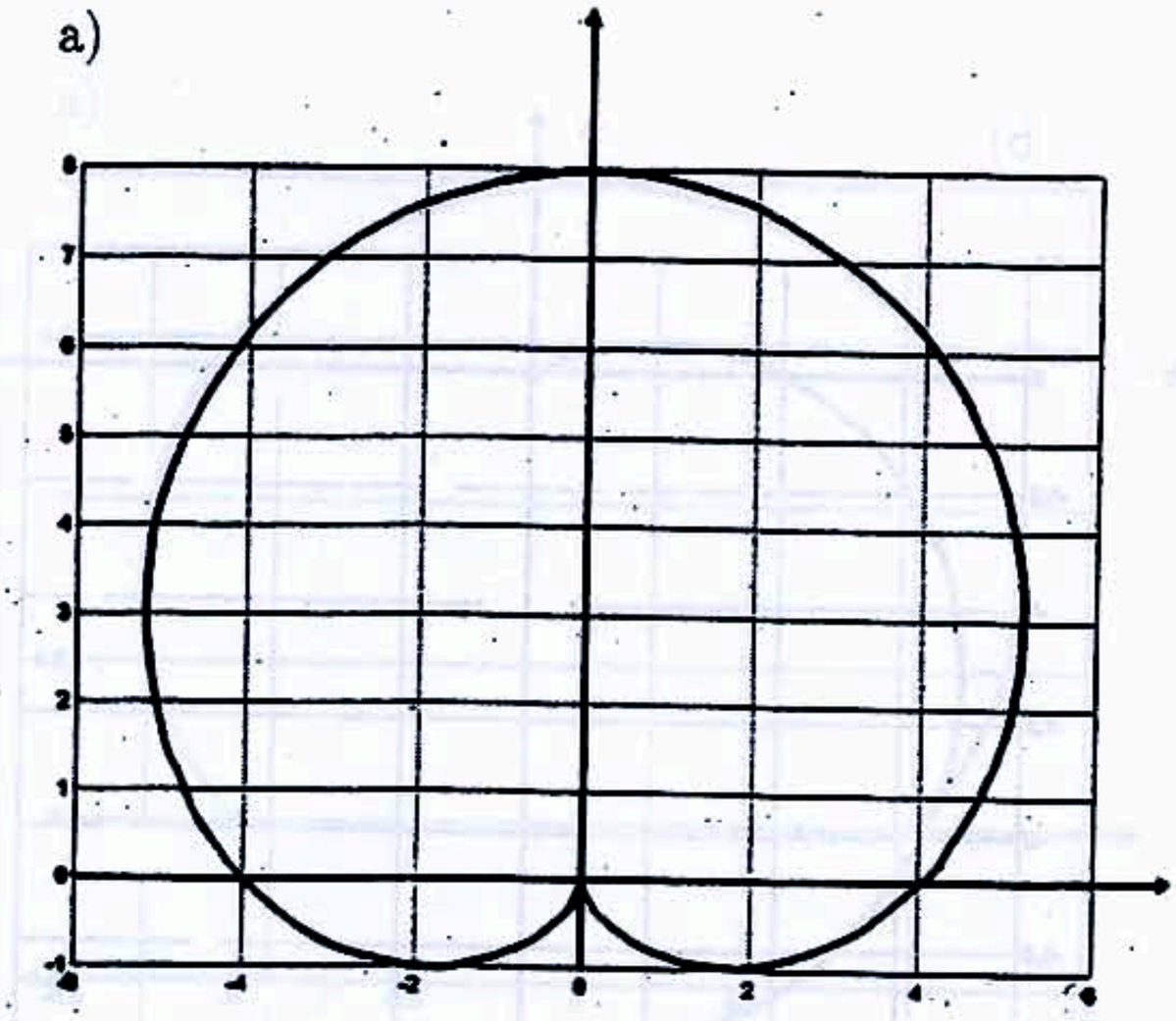


d)

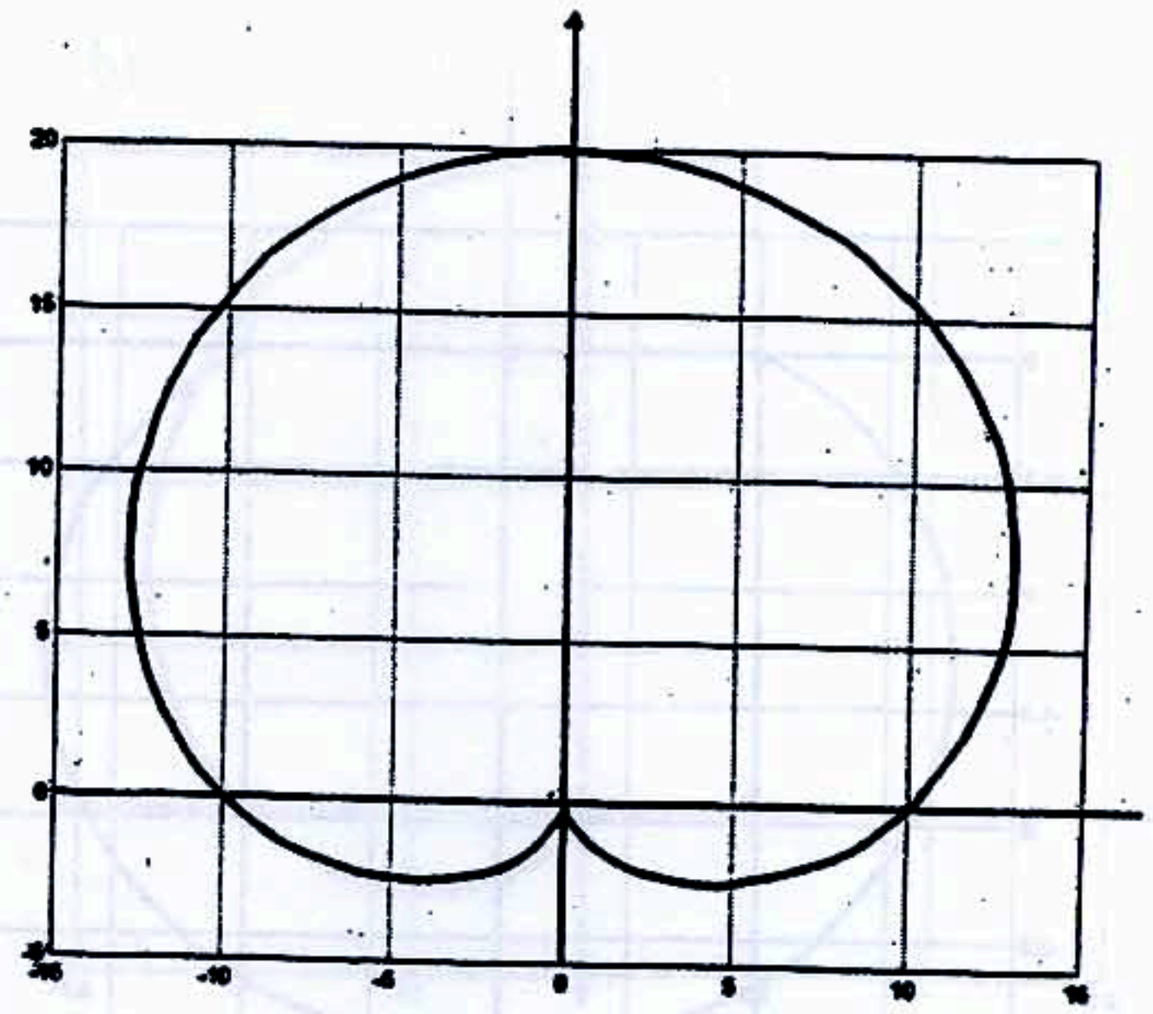


15. Below are the graphs of some polar curves of the form $r = a(1 + \sin\theta)$. What is the value of a in each case?

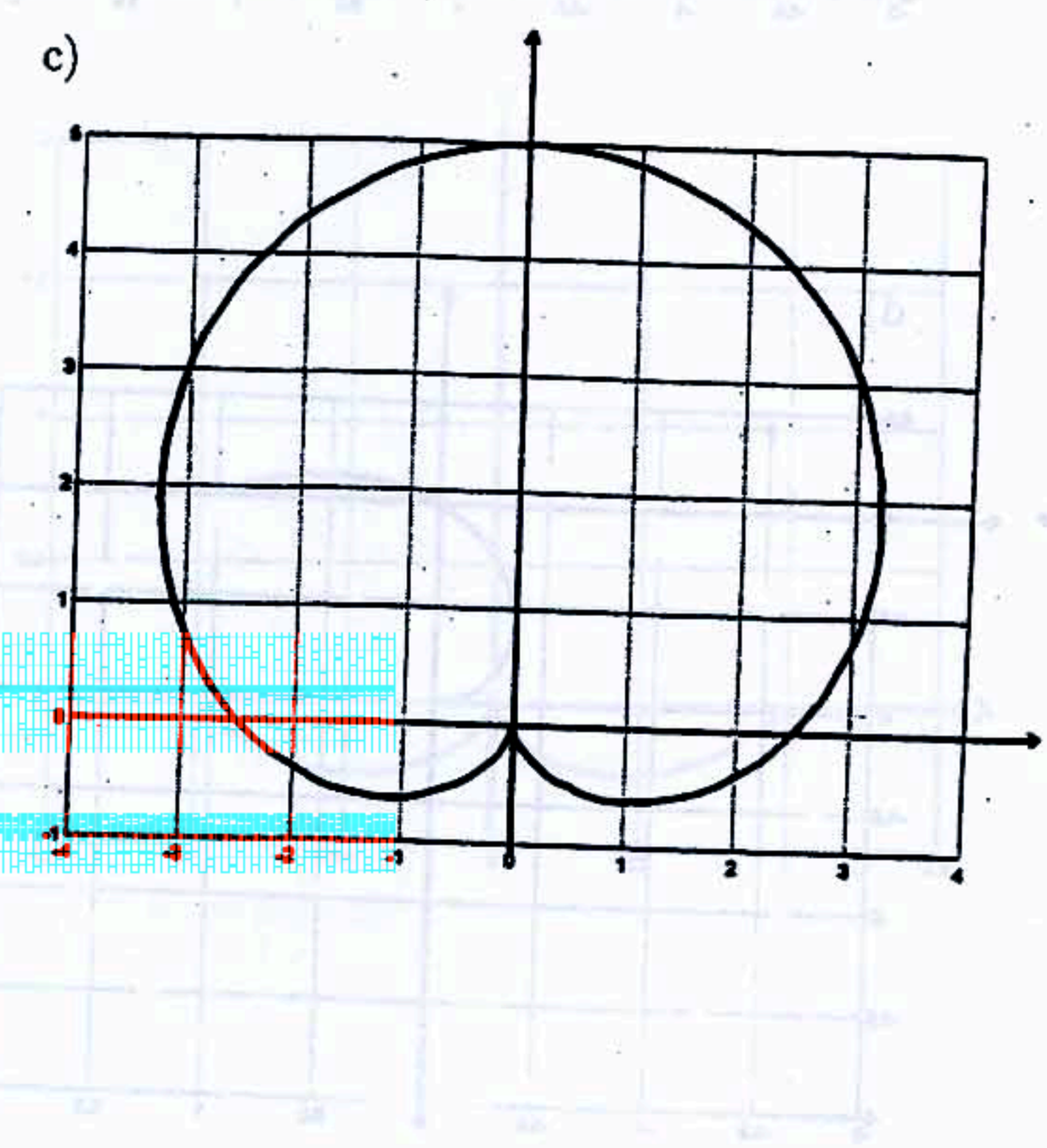
a)



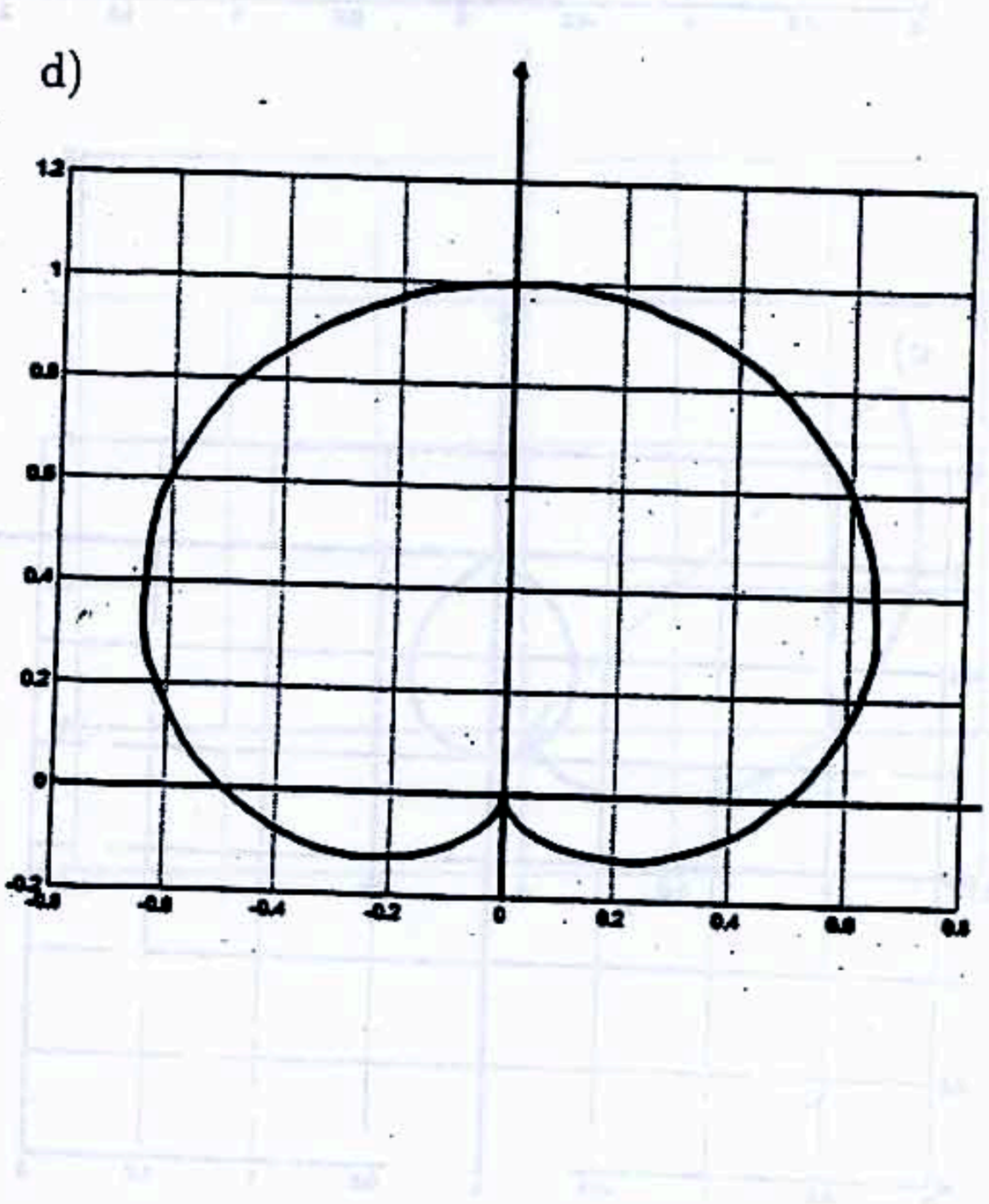
b)



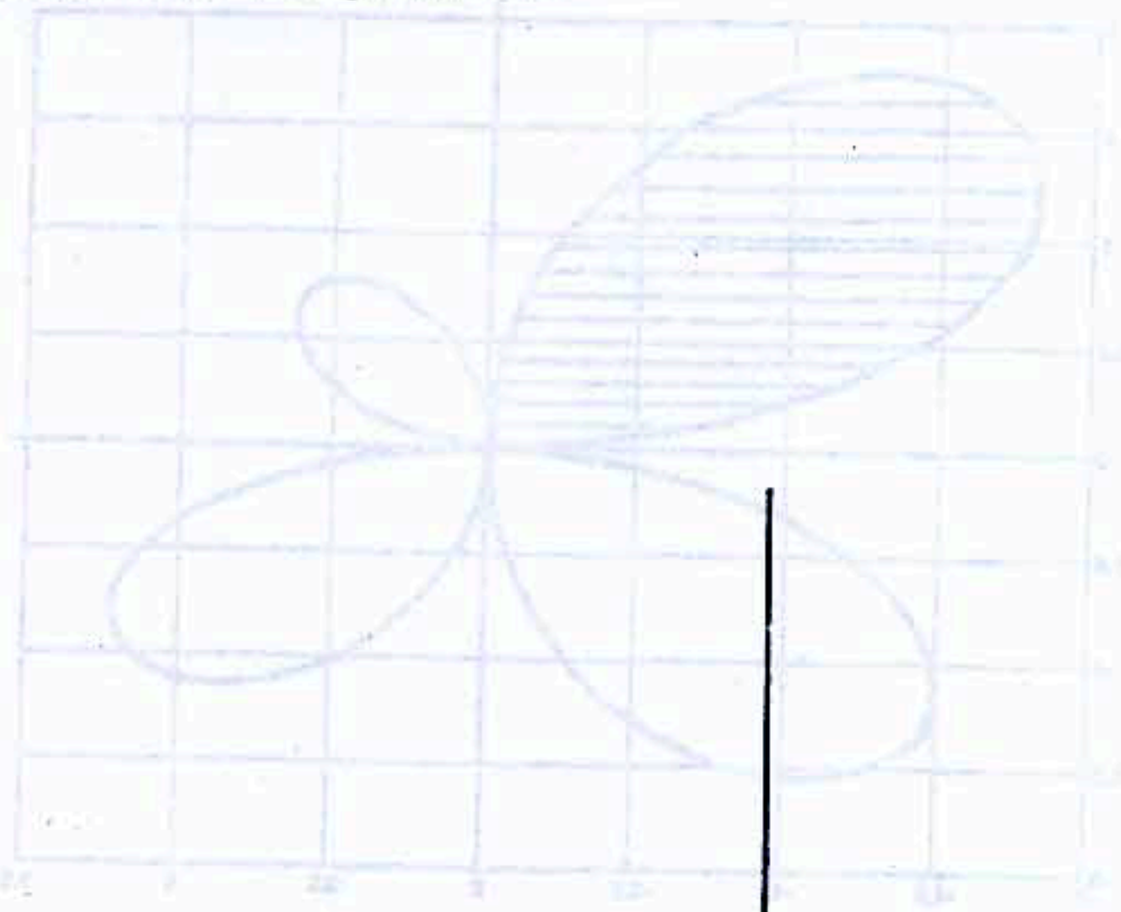
c)

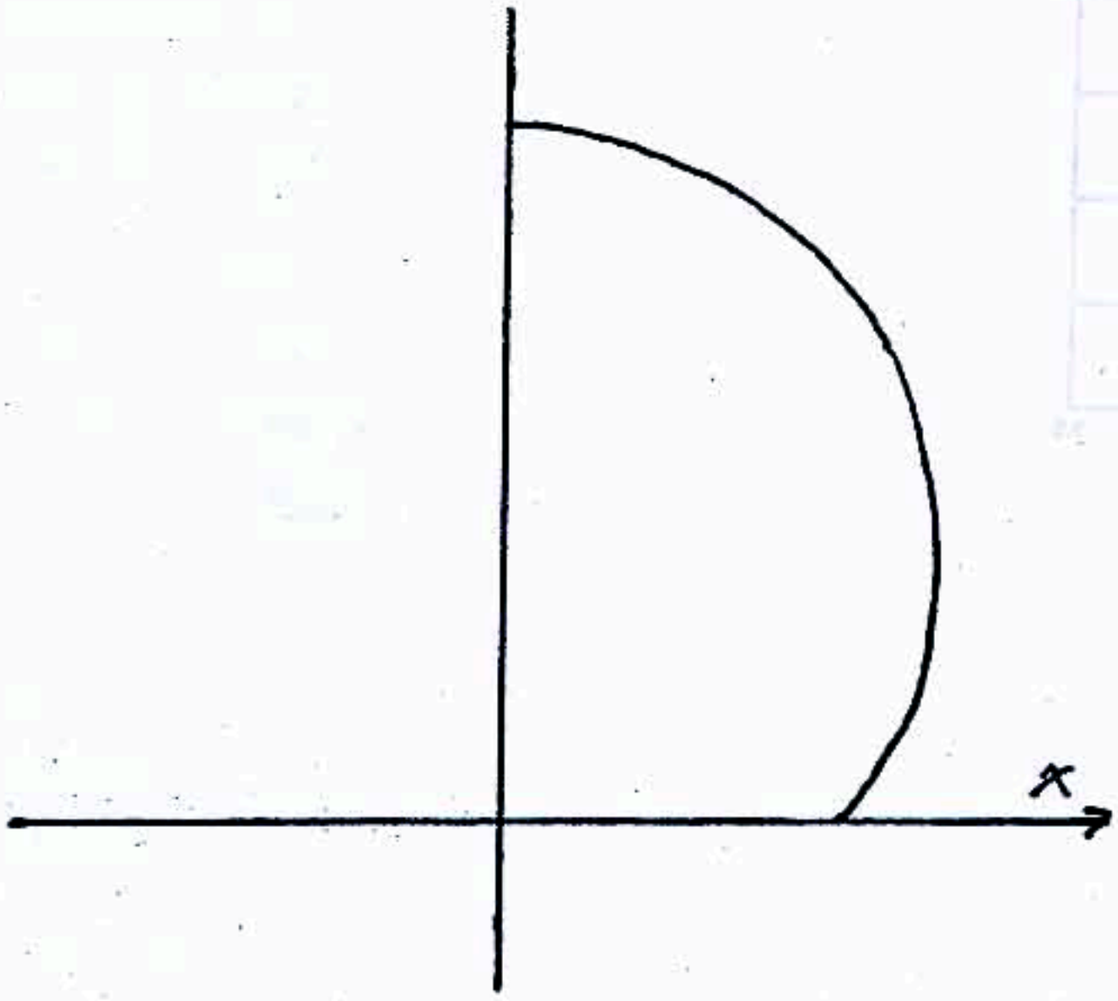


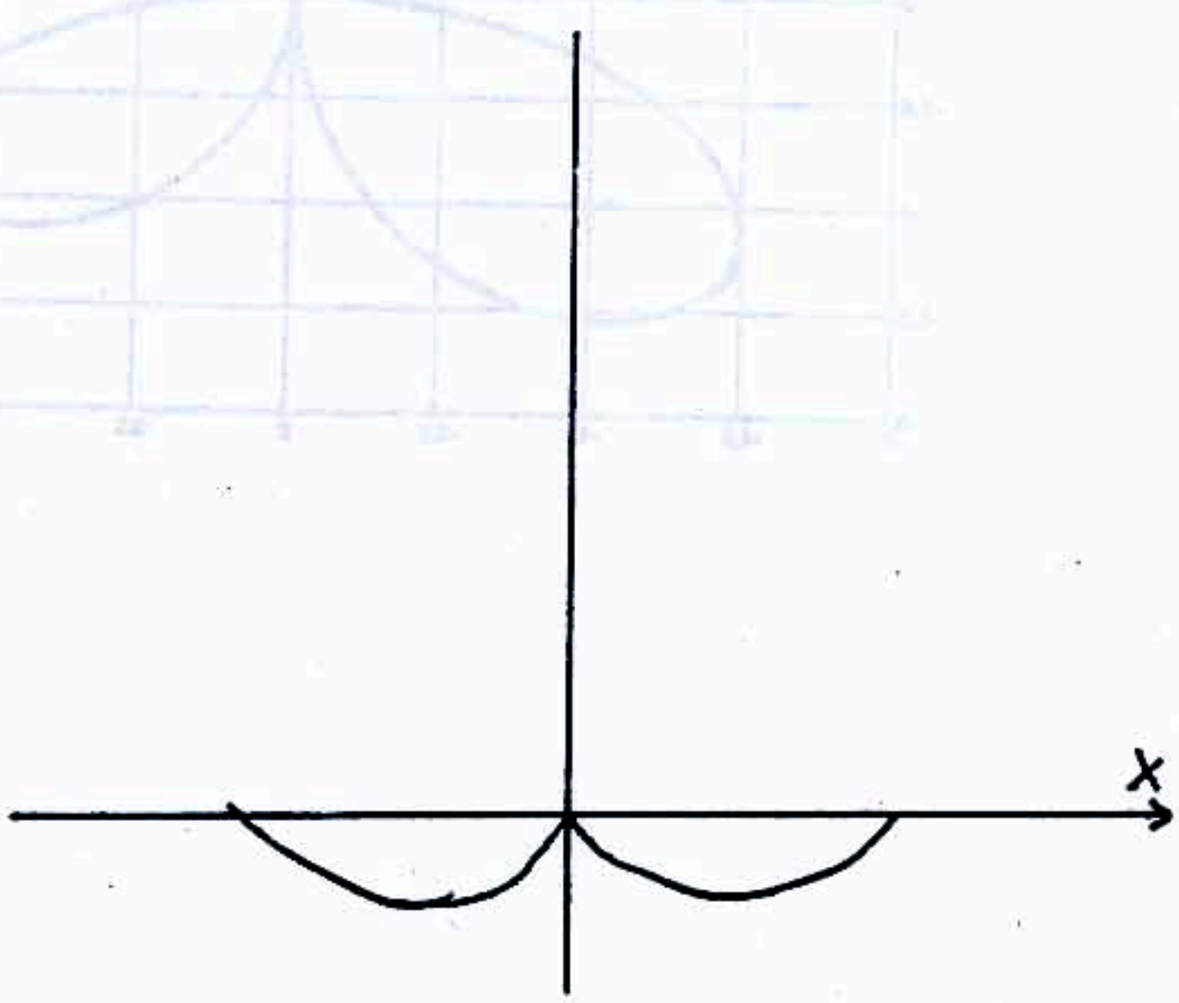
d)

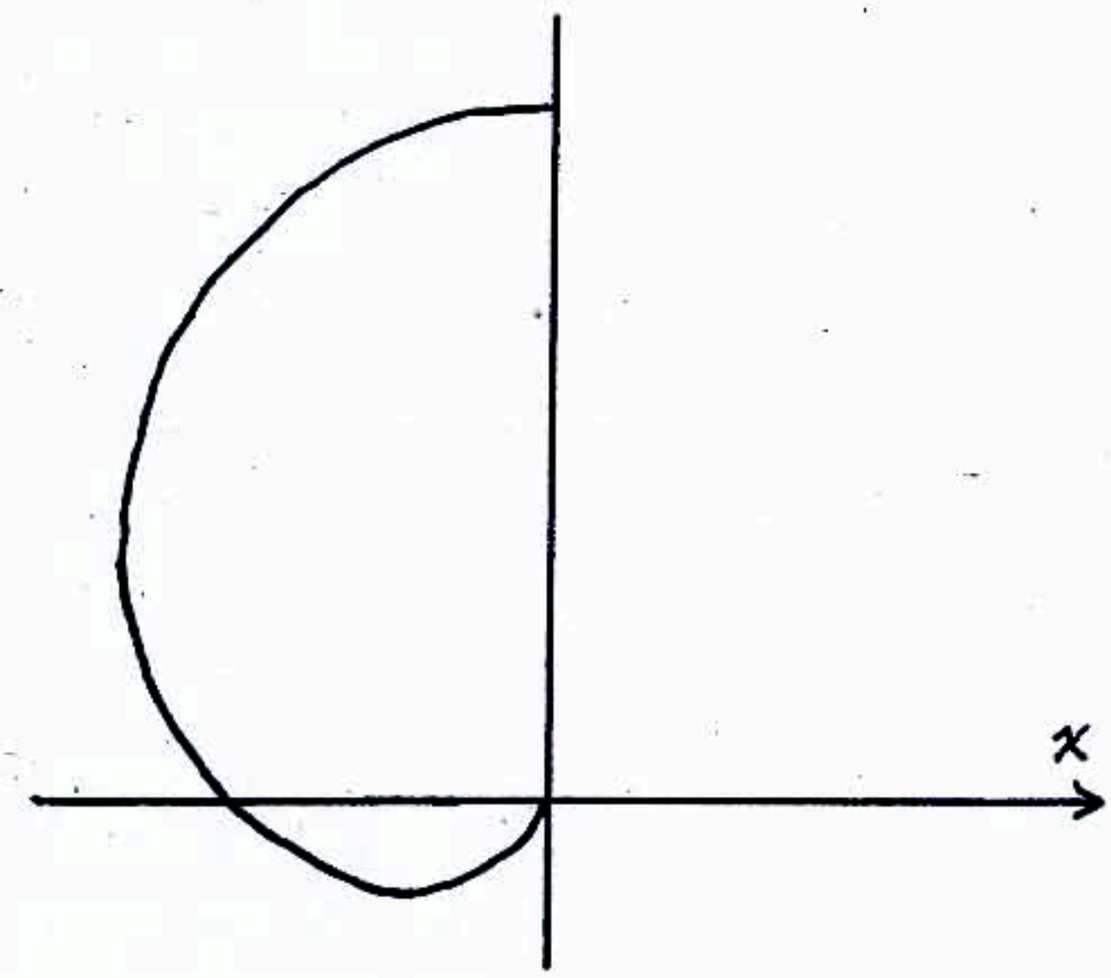


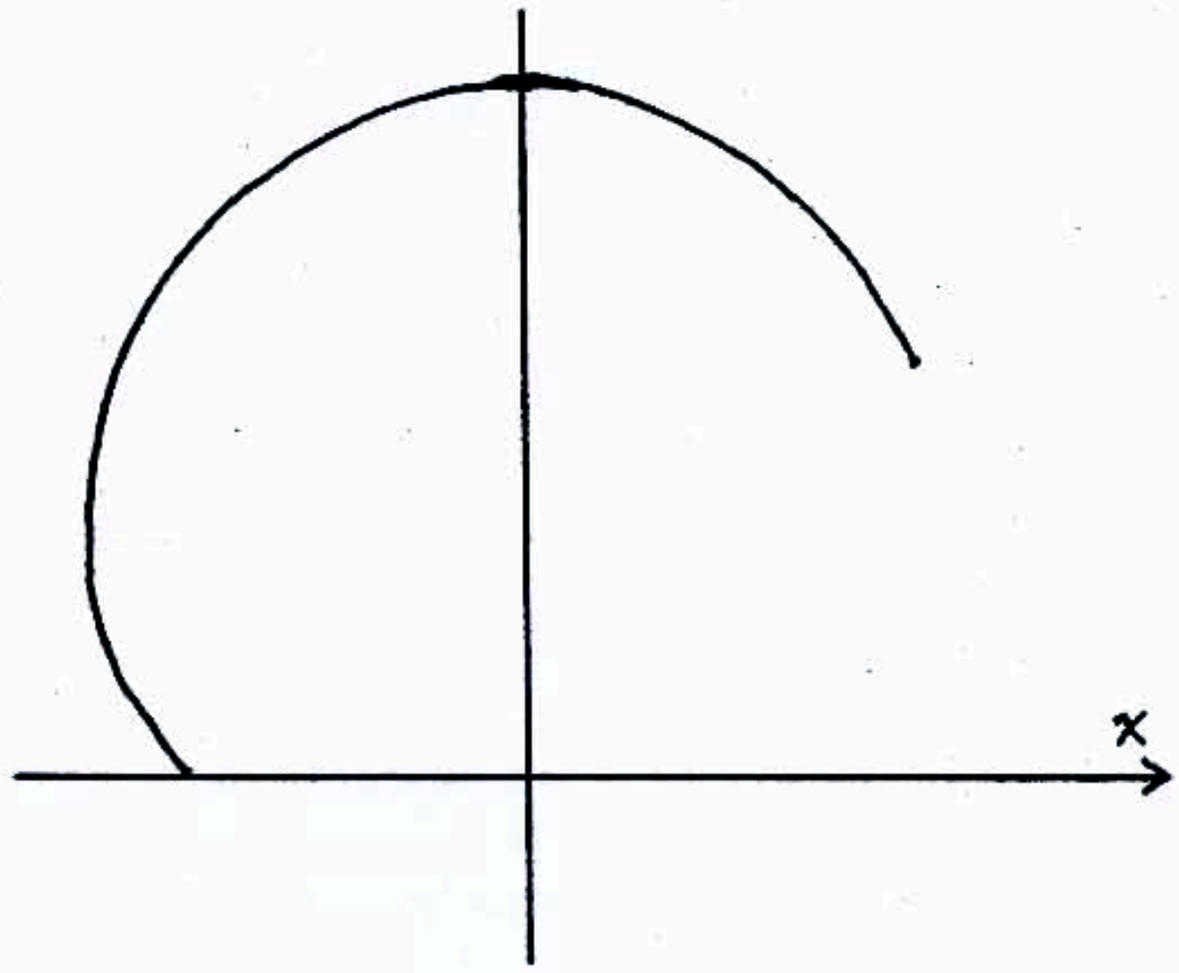
16. Below are some parts of the graph of the polar equation $r = \sin\theta - 1$. State exactly the values of θ which correspond to the sections of the graph shown.











17. Find the area of the shaded leaf of the four leaved rose $r = \sqrt{\theta}\sin 2\theta,\ \theta \geq 0$.