6981 Special Problems

1. Find each of the following general antiderivatives using an appropriate u-substitution.

a) $\int \frac{4x+12}{x^2+16}dx$ b) $\int \frac{xdx}{\sqrt{25-16x^2}}$

c) $\int x\sqrt{3x+5}dx$ d) $\int \frac{(x+4)dx}{(x^2+8x+15)^{3/2}}$

2. Evaluate the following indefinite integrals using integration by parts.

a) $\int x\sqrt{3x+5}dx$ b) $\int x^3(\ln x)dx$

c) $\int x^2e^{3x}dx$

3. Make use of some well known trigonometric identity to find the following antiderivatives.

a) $\int [\tan 3x]^4[\sec 3x]^4dx$ b) $\int [\sin(3x)]^4dx$

4. Evaluate the following indefinite integral using the substitution $x = (5/2)\sin\theta$.

$$\int \frac{x^2dx}{\sqrt{25-4x^2}}$$

5. Evaluate the following indefinite integral using the substitution $x = (5/4)\tan\theta$.

$$\int \frac{x^2dx}{25+16x^2}$$

6. Evaluate the following indefinite integral using a substitution involving either $\sin\theta$ or $\tan\theta$.

$$\int x^3\sqrt{16-9x^2}dx$$

7. Evaluate the following indefinite integral using a substitution involving either $\sin\theta$ or $\tan\theta$.

$$\int \frac{x^3}{\sqrt{16-9x^2}}dx.$$

8. Evaluate the following indefinite integral using the substitution $x = 3 + (4/3)u$.

$$\int \frac{dx}{9x^2-54x+97}$$

9. Find an approximate value of the definite integral $\int_0^5 (5x - x^2)dx$ using the trapezoid rule with $n = 5$.

10. Find an approximate value of the definite integral $\int_2^5 \frac{x}{1+x}dx$ using the trapezoid rule with $n = 6$.

11. Find numbers A and B such that

$$\frac{A}{x+4} + \frac{B}{x-7} = \frac{13x-3}{(x+4)(x-7)}.$$

12. Find numbers A and B such that

$$\frac{A}{x+5} + \frac{B}{x-2} = \frac{x-9}{(x+5)(x-2)}.$$

13. Find numbers A, B, and D such that

$$\frac{A}{x+9} + \frac{Bx+D}{x^2+4} = \frac{8x^2+37x+110}{(x^2+4)(x+9)}.$$

14. Convert the following definite integral to another definite integral of equal value with variable of integration u using the substitution $u = x + 3$. You need not evaluate the resulting integral

$$\int_1^6 \frac{x}{\sqrt{x+3}}dx.$$

15. Convert the following integral to another definite integral of equal value with variable of integration u using the substitution $u = 5x + 6$. You need not evaluate the resulting integral

$$\int_2^6 x\sqrt{5x+6}dx$$

16. Convert the following definite integral to another definite integral of equal value with variable of integration θ using the substitution $3x = 5\sin\theta$. You need not evaluate the resulting integral

$$\int_0^{5/6} \frac{x^2dx}{\sqrt{25-9x^2}}$$

17. Convert the following definite integral to another definite integral of equal value with variable of integration u using the substitution $x = (5/3)u$.

$$\int_0^{5/6} \frac{dx}{\sqrt{25-9x^2}}$$

9. Find an approximate value of the definite integral $\int_0^5 (5x - x^2)dx$ using the trapezoid rule with $n = 5$.

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16. Convert the following definite integral to another definite integral of equal value with variable of integration θ using the substitution $3x = 5\sin\theta$. You need not evaluate the resulting integral

$$\int_0^{5/6} \frac{x^2dx}{\sqrt{25-9x^2}}$$

17. Convert the following definite integral to another definite integral of equal value with variable of integration u using the substitution $x = (5/3)u$.

$$\int_0^{5/6} \frac{dx}{\sqrt{25-9x^2}}$$

18. Convert the following definite integral to another definite integral of equal value with variable of integration u using the substitution $x = e^u$.

$$\int_1^e x^3(\ln x)^2 dx$$

19. Convert the following definite integral to another definite integral of equal value with variable of integration θ using the substitution $x = (5/3)\tan\theta$.

$$\int_{5/3}^{5/\sqrt{3}} \frac{x^2 dx}{9x^2+25}$$

20. The formula for the error bound in the trapezoidal rule is

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}.$$

Suppose $|f''(x)| \leq 15$ for all x. Find the maximum error when we approximate the integral

$$\int_2^5 f(x)dx$$

using the trapezoidal rule with $n = 27$.

21. Suppose $|f''(x)| \leq 5$ for $1 \leq x \leq 7$. Find the maximum possible error made when we approximate the integral $\int_1^7 f(x)dx$ using the trapezoid rule with $n = 100$.

22. Find an error bound, a maximum value for the error, when we approximate the integral $\int_0^5 x^3 dx$ using the trapezoid rule with $n = 50$.

23. Suppose we are estimating the integral $\int_2^8 f(x)dx$ using the trapezoid rule. Given that $|f''(x)| \leq 12$ for all x, what is the smallest value we can choose for n and still be sure that $|E_T| \leq 10^{-4}$?

24. Find the maximum error or error bound when we approximate the following integral using the trapezoidal rule with $n = 30$:

$$\int_1^6 (10x - x^2)dx.$$

25. Suppose $|f''(x)| \leq 12$ for all x. Find the smallest value of n such that $|E_T| \leq 10^{-4}$ when we approximate the following integral using the trapezoidal rule:

$$\int_2^6 f(x)dx.$$

26. Find the indefinite integral $\int \frac{x^3+8}{x^2+16}dx$.

27. Evaluate the indefinite integral

$$\int \frac{8x^2+37x+110}{(x^2+4)(x+9)}dx.$$

28. Evaluate the indefinite integral

$$\int \frac{8x^2+21x+72}{(x^2+9)(x^2+16)}dx.$$

29. Find the arc length of the curve which is the graph of $f(x) = (1/6)(x^2+16)^{3/2}$ for $0 \leq x \leq 6$.

30. Find the arc length of the curve which is the graph of $f(x) = (1/4)x^4 + (1/8)x^{-2}$ for $1 \leq x \leq 5$.

31. Find the arc length of the curve which is the graph of $f(x) = (1/3)x^{3/2} - x^{1/2}$ for $0 \leq x \leq 4$.

32. Find the surface area of the surface obtained by rotating the curve $y = \sqrt{x}$ for $0 \leq x \leq 9$ about the x axis.

33. First, explain why each of the following integrals is improper. If the improper integrals is convergent, then evaluate it. If the integral is divergent, then explain why it is divergent.

a) $\int_0^5 \frac{xdx}{\sqrt{25-x^2}}$ b) $\int_{-1}^1 \frac{dx}{x+1}$

c) $\int_0^\infty \frac{xdx}{\sqrt{25+x^2}}$ d) $\int_0^\infty \frac{xdx}{(25+x^2)^{3/2}}$

34. The region bounded by the parabola $y = 6x - x^2$ and the line $y = x$ is covered by a lamina of constant mass density ρ. Find M_x, the moment about the x axis. Find the mass of the lamina.

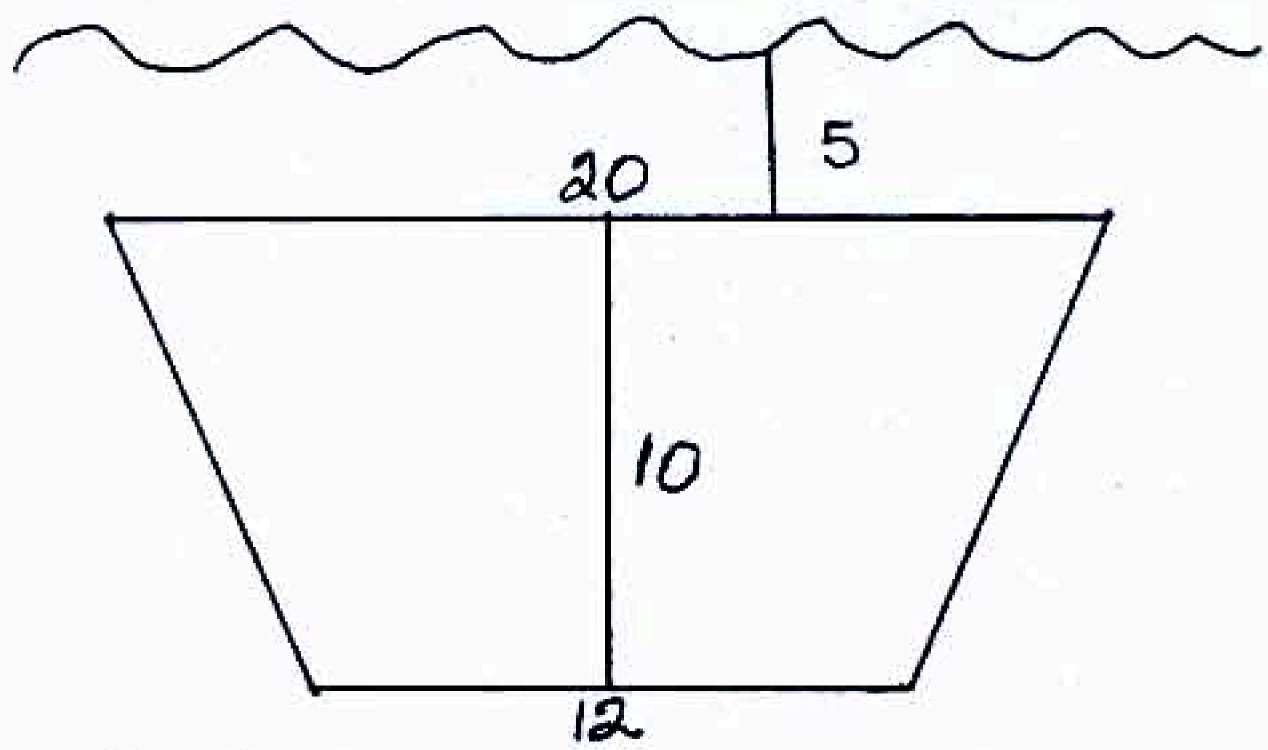
35. The region bounded by the parabola $y = (x-2)^2$ and the line $y = 4$ is covered by a lamina of mass density ρ. Find M_x, the moment about the x axis.

36. The region bounded by the parabola $y = 6x - x^2$ and the line $y = -x$ is covered by a lamina of constant density ρ. Find M_y, the moment of this lamina about the y axis. Find the mass of the lamina.

37. The region bounded by the parabola $y = 6x - x^2$ and the line $y = x$ is covered by a lamina of variable density $\rho(x) = x$. Find M_y, the moment of this lamina about the y axis.

38. Let R denote the region bounded by the curve $y = \sin x$, the x axis, the line $x = 0$, and the line $x = \pi/2$. The region R is covered by a lamina of constant density ρ. Find M_x, the first moment about the x axis, and M_y, the first moment about the y axis, for this lamina.

39. A vertical plate forms part of a large container. The container is filled with water. The top edge of the plate is 5 feet below the water level (surface). The plate has the shape of a trapezoid and is 20 feet across the top, 12 feet along the bottom, and 10 feet from top to bottom. Find the hydrostatic force on the plate.



40. The equations $y = x^2 - c^2$, where c is an arbitrary constant defines a family of functions. There is a function in the family for every value of c. Sketch the functions of the family obtained using $c = 0, 1, 4,$ and 9 on the same set of axis.

41. Find the general solution of each of the following differential equations.

a) $\dfrac{dy}{dx} = 2x + \dfrac{1}{x^2+1}$ b) $\dfrac{dy}{dx} = \dfrac{2(y+5)}{x+4}$

c) $\dfrac{dy}{dx} = \dfrac{2(y^2+10)}{x+5}$ d) $\dfrac{dy}{dx} = \dfrac{3y}{2(x+5)}$

42. A tank contains 25 kg of salt dissolved in 4,000 L of water. Brine that contains (1/10) kg of salt per liter of water enters the tank at the rate of 20 L/min. Brine is drained from the tank at the same rate of 20 L/min. Find an expression for the amount of salt in the tank at time t.

43. A tank contains 80 lbs of salt dissolved in 600 gallons of water. Brine that contains (1/3) lb of salt per gallon of water enters the tank at the rate of 15 gal/min. Brine is drained from the tank at the same rate of 15 gal/min. Find an expression for the amount of salt in the tank at time t.

44. A tank contains 40 kg salt dissolved in 6,000 L of water. Brine that contains (1/6) kg of salt per liter of water enters the tank at the rate of 24 L/min. Brine is drained from the tank at the same rate of 24 L/min. Find an expression for the amount of salt in the tank at time t.

45. Solve the initial value problem:

$$\frac{dy}{dt} = 6 - y \text{ and } y(0) = 3.$$

46. Solve the initial value problem:

$$\frac{dy}{dt} = y(y - 8) \text{ and } y(0) = 2.$$

47. Solve the initial value problem:

$$\frac{dy}{dt} = y^2 + 16 \text{ and } y(0) = 4.$$

48. Solve the initial value problem:

$$\frac{dy}{dt} = (2 + y)(6 - y) \text{ and } y(0) = 4.$$

49. Find y_0, y_1, y_2, y_3, y_4 and y_5 for the initial value problem $y' = 2x + 3y$ and $y(1) = 2$ using the Euler method with step size $h = 0.2$.