Math 2243, Midterm Exam 1

October 4, 2001

INSTRUCTIONS: Books and notes are not allowed. Calculators are allowed. Write *complete solutions* to all problems for full credit. You have 60 minutes to work on the problems.

Name: ______________________ TA Section: ______________________

1) (15 pts) Find the equation of the orthogonal trajectories to the family of curves $y^2 = 3x + c$.

2) (*a*) (10 pts) Find the general solution to the equation $y' \cos x + y \sin x = \sin x$.
(*b*) (10 pts) Solve the IVP $\quad xy' - 2y = 2x^2 \ln x, \quad y(1) = 3.$

3) Find the general solution for each of the equations:

(a) (6 pts) $y'' - y' + 6y = 0$ (b) (13 pts) $y'' + 10y' + 25y = e^{-5x}$

(c) (14 pts) $y'' + 2y = 17e^{-x} \sin 2x$

4) Consider the equation $x^2y'' - 3xy' + 4y = 0$.

(a) (6 pts) Find a solution of the form $y_1(x) = x^r$.

(b) (11 pts) Use the method of reduction of order to find another solution $y_2(x)$.

(c) (5 pts) Show that y_1 and y_2 are linearly independent by calculating their Wronskian, and write the general solution to the equation.

5) (20 pts) Recall that the differential equation governing an RL circuit is $\frac{di}{dt} + ai = \frac{1}{L}E(t)$, where $i(t)$ is the current in the circuit at time t and $a = \frac{R}{L}$. Consider an RL circuit with zero initial current and electromotive force $E(t) = E_0 e^{bt}$. Find the current in the circuit for $t > 0$. (Caution: At some point you will need to treat the cases $b \neq -a$ and $b = -a$ separately.)