Math 2243, Midterm Exam 2

November 8, 2001

INSTRUCTIONS: Books and notes are not allowed. Calculators are NOT allowed. Problems 1-3 are in "multiple choice" format. For these problems circle the answer you believe to be correct. Write *complete solutions* to problems 4-7 for full credit. You have 60 minutes to work on the problems.

Name: ______________________ TA Section: ______________________

1) (10 pts) Which of the following is a subspace of the given vector space

(a) $\{(x_1, x_2 \dots x_n) \in \mathbb{R}^n \mid x_1 + x_2 + \cdots x_n = 1\}$
(b) The subset of all polynomials of degree exactly 3 in the vector space $\mathcal{P}_4$ of all polynomials of degree at most 3.
(c) The subset of all matrices in $M_3(\mathbb{R})$ which have a zero in the upper right corner, that is

$$\left\{ A \in M_3(\mathbb{R}) \mid A = \begin{bmatrix} a & b & 0 \\ c & d & e \\ f & g & h \end{bmatrix} \right\}$$

(d) $\{y \in C^2((a,b)) \mid y'' + y' + 3y = \cos x\}$

2) (10 pts) What is the maximum possible number of vectors in a linearly independent subset of the vector space $M_3(\mathbb{R})$ of 3×3 matrices with real entries?

(a) 3
(b) 9
(c) there is no maximum number
(d) 4

3) (10 pts) Let A be an $m \times n$ matrix and let $\mathbf{b}$ be a column n-vector. The linear system $A\mathbf{x} = \mathbf{b}$ (for which $A^{\#}$ denotes the augmented matrix) has a unique solution if

(a) $\text{rank}(A) = \text{rank}(A^{\#})$
(b) $\text{rank}(A) < \text{rank}(A^{\#})$
(c) $\text{rank}(A) = \text{rank}(A^{\#}) = n$
(d) $\text{rank}(A) = \text{rank}(A^{\#}) = m$

End of multiple choice problems

4) (20 points) Consider the matrices

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

Calculate, if possible, $(A - 2B)C$, $(AB - 2A)C$, and $A^TB + AB^T$. If one (or more) of these expressions is not defined, state so and give the reason.

5) (*a*) (20 points) Calculate $\det(A)$ where A is the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 1 & -1 & 1 \end{bmatrix}.$$

Is the matrix A nonsingular? Why or why not?

(*b*) The system

$$A\mathbf{x} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution (x_1, x_2, x_3). Find x_3 using Cramer's Rule.

6) (25 pts) (a) Use the method of Gaussian elimination to solve the system of equations $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & -1 & 0 & -1 \\ 2 & 1 & 3 & 7 \\ 3 & -2 & 1 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

(b) Find a spanning set for the nullspace of A. (Recall that this is the subspace of $\mathbb{R}^4$ defined as $\text{Nullspace}(A) = \{\mathbf{x} \in \mathbb{R}^4 \mid A\mathbf{x} = \mathbf{0}\}$.) Is the spanning set you found a basis for Nullspace(A)? Justify.

7) (15 pts) Show that the vectors $\mathbf{v}_1 = (1, 1, 1, 1)$, $\mathbf{v}_2 = (1, 2, 3, 0)$, $\mathbf{v}_3 = (3, 6, 0, 0)$ and $\mathbf{v}_4 = (-1, 0, 0, 0)$ form a basis of $\mathbb{R}^4$. (Hint: the determinant of the matrix $[\mathbf{v}_1\mathbf{v}_2\mathbf{v}_3\mathbf{v}_4]$ is particularly easy to calculate if you choose a "good" row, or column, along which you exapnd it.)