Math 2243, Final exam.

Name:_____ Instructor:_____

Remember to show all your work. Without it, a correct answer may be given no credit.

(1) **Problem 1**.

Find the general solution $y(t)$ to the following ODE: $y' + 3ty = t$.

(2) **Problem 2**.

Show that the functions $\{e^t, te^t, t^2 e^t\}$ are linearly independent by using the Wronskian.

(3) **Problem 3.**

Find the solution to the initial value problem $y'' - y = 4e^t$, $y(0) = -1$, $y'(0) = 1$.

(4) **Problem 4.**

Find the general form of the solution for the following linear system

$$x' = y \ ; y' = x$$

(5) **Problem 5**.

Let $K \subset \mathbb{R}^3$ be the subspace $K = \{(x, y, z) \in \mathbb{R}^3 | x = y\}$ and let $I \subset \mathbb{R}^3$ be the subspace $I = \{(x, y, z) \in \mathbb{R}^3 | x = -y \text{ and } z = 0\}$. I.e., K consists of vectors of the form (x, x, z) and I consists of vectors of the form $(x, -x, 0)$.

Find a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which has K as its kernel and L as its range.

(6) **Problem 6.**

Find a 2x2 matrix A with non-zero entries which satisfies the equation
$$A^2 + A = 0.$$