Midterm Exam III 1 2 3 4

Last Name:_____

ID:_____ Section:____

Math 2243, April 28, 2004

There are 4 partial credit questions. NO GRAPHIC CALCULATORS are permitted. GOOD LUCK !

- 1. Answer all the following questions :
- a) Is there any injective linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$. If yes, give an example. [5 pts.]
- b) Give an example of a matrix satisfying the equation $A^2 + 2A + I_2 = 0.$ (Hint: remember that every matrix satisfyes its own characteristic equation). [5 pts.]
- c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection about the line of equation x = y. Find the eigenspaces of T. [15 pts.]

- 2.
- a) Using the conservation of energy principle(kinetic+potential=constant) write down the equation of motion for a simple pendulum(interpreting the angle made the pendulum with the vertical equilibrium postion as of function of time).DO NOT ATTEMPT TO SOLVE THE EQUATION YOU GET![10 pts.]
- b) Is the equation you obtain in part a) a linear differential equation? [10 pts.]

3. Solve the Initial-Value-Problem

$$y'' + y' = e^{-t}; y(0) = 0; y'(0) = 0$$

(Hint: try a particular solution $y_p(t) = te^{-t}$) [35 pts.]

4. Let $v_0 = <1, 0, -1 > \in \mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined as

$$T(v) = v \times v_0.$$

What is the kernell and the range of this linear transformation?[15 pts.] What is the matrix of the linear transformation?[10 pts.]