MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
SAMPLE MIDTERM TEST III

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. You must show all work.
Good luck!

Problem 1.   (1) Find the inverse of the following matrix:
$$A = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

(2) Use the inverse of A to solve the system (another way of solving it will not be counted)
$$\begin{aligned} 2y + z &= 1, \\ x + z &= 0, \\ x - y &= -1. \end{aligned}$$

Problem 2. Use Cramer's rule to determine the unique solution to the system $A\mathbf{x} = \mathbf{b}$ for the following matrix and vector:
$$A = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 5 \\ 2 & 3 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix}.$$

Problem 3.   (1) Determine whether or not the set
$$S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$$
is a subspace of $\mathbb{R}^2$. Justify your answer.
(2) What is the span of this subset? Justify your answer.

Problem 4. Decide whether the set $S = \{x^2, 1 - x^2\}$ is a linearly independent subset of the vector space P_3 of polynomials of degree less than three. Justify your answer.

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