# MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS SAMPLE MIDTERM TEST IV 

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. You must show all work.

Good luck!
Problem 1. Let $V=\left\{(x, y, z) \in \mathbb{R}^{3}: x+y-z=0\right\}$. Find a basis of $V$.
Problem 2. Let $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ be the reflection about the line $y=x$. Find the matrix $A$ of the linear transformation $T$ and the eigenspaces of $A$.

## Problem 3.

Determine whether the matrix

$$
A=\left[\begin{array}{cc}
1 & -3 \\
-2 & 2
\end{array}\right]
$$

is diagonalizable. If it is, find a matrix $S$ that diagonalizes $A$ and determine $S^{-1} A S$.
Problem 4. Solve the IVP:

$$
\begin{aligned}
\mathbf{x}^{\prime} & =\left[\begin{array}{cc}
1 & -3 \\
-2 & 2
\end{array}\right] \mathbf{x}, \\
\mathbf{x}(\mathbf{0}) & =\left[\begin{array}{c}
1 \\
-1
\end{array}\right]
\end{aligned}
$$

Problem 5. Determine the general solution to the system $\mathbf{x}^{\prime}=A \mathbf{x}$ for

$$
A=\left[\begin{array}{lll}
2 & -1 & 3 \\
2 & -1 & 3 \\
2 & -1 & 3
\end{array}\right]
$$

