(1) **Problem 1**.

Find the general solution $y(t)$ to the following ODE: $y' + y = 1$.

(2) **Problem 2**.

Show that the functions $\{t^3, te^t, t\}$ are linearly independent by using the Wronskian. Is t^2 spanned by the previous functions?

(3) **Problem 3.**

Find the solution to the initial value problem $y''+y = e^t$, $y(0) = -1$, $y'(0) = 1$.

(4) **Problem 4**.

Find the general form of the solution for the following linear system

$$x' = x + y \ \ ; y' = x$$

(5) **Problem 5**.

Let T be the the rotation of angle θ of the two dimensional plane R^2. This is a linear transformation. Determine its matrix and find all the values of θ for which this linear transformation admits at least one real eigenvalue.

(6) **Problem 6.**

Find a 2x2 matrix A satysfing

$$A^2 - 3A = -2I_2.$$

(7) **Problem 7.**

a) Give an example of two square matrices A and B such that $AB \neq BA$
b) Find a nonzero matrix(a matrix for which not all the entries are zero) that satisfies the equality $A^2 = 0$

(8) **Problem 8.**
Let $V = [(x, y, z) \in R^3 | x + y - z = 0]$. Find two linearly independent vectors in V.

(9) **Problem 9.**

Answer the following two questions :
a) Verify if the Picard conditions are satisfied for the following initial value problem: $y' = y^t y$, and $y(10) = 10$.
b) Construct(if possible!) a differential equation that has only one stable(sink) equilibrium.Draw the phase-line for this equation.

(10) **Problem 10.**

A certain radioactive substance has a half-life of 5 hours. Find the time for a given amount to decay to one-eight of its original mass.

(11) **Problem 11.**

At noon , with the temperature in your hous at 75^oF and the outside temperature at 95^oF, your air conditioner breaks down. Suppose the time constant $1/k$ for your house is 4 hours. when will the temperature in your house reach 80^oF ?

(12) **Problem 12.**

Solve the first-order differential equation (*by the method you prefer!*):

$$y' - y = \frac{y}{t^2+1}$$

(13) **Problem 13.**
Answer all the following questions :
a) Give an example of a matrix satisfying the equation $A^2+2A+I_2 = 0$.(Hint: remember that every matrix satisfyes its own characteristic equation).
b) Let $T : R^2 \to R^2$ be the reflection about the line of equation $x = 2y$.Find the eigenspaces of T.

(14) **Problem 14.**

a) Using the conservation of energy principle(kinetic+potential=constant) write down the equation of motion for a simple pendulum(interpreting the angle made the pendulum with the vertical equilibrium postion as of function of time).DO NOT ATTEMPT TO SOLVE THE EQUATION YOU GET!
b) Is the equation you obtain in part a) a linear differential equation?

(15) **Problem 15.**
Solve the Initial-Value-Problem

$$y" + y' = e^{-t}; y(0) = 0; y'(0) = 0$$

(16) **Problem 16.**
Let $v_0 =< 1, 0, -1 >\in R^3$ and $T : R^3 \to R^3$ be the linear transformation defined as

$$T(v) = v \times v_0.$$

Determine the matrix associated to this linear transformation.

(17) **Problem 17.**

Solve the differential equation

$$y' = t^2 e^{y+2t}$$

(18) **Problem 18.**

Show that the initial value problem

$$y' = y^{1/3}, \;\; y(0) = 0$$

exhibits nonunique solutions and sketch graphs of several possibilities. What does Picard's theorem tell you for this problem?

(19) **Problem 19.**
The DE

$$y' = \frac{y^2}{e^y - 2ty}$$

looks impossible to solve analytically, but treating y as the independent variable and t as the dependent variable, an implicit solution can be found. Carry out this solution.

(20) **Problem 20.**

Into a tank containing 100 gal od fresh water, Bob was to have added 10 lbs of salt but accidentally added 20 lbs instead. To correct his mistake he started adding fresh water at a rate of 3 gal/min, while drawing off well-mixed solution at the same rate. How long will it take until the tank contains the correct amount of salt?

(21) **Problem 21.**

Suppose A and B are $n \times n$ matrices and AB and B are invertible. Show that A must be invertible using matrix algebra.

(22) **Problem 22.**
Show that the set of integers with the standars operations is not a vector space by identifying at least one vector space property that fails.

(23) **Problem 23.**
Solve the differential equation

$$y" - 3y' = 2y = e^t \sin(t)$$

(24) **Problem 24.**
Use the Cayley-Hamilton Theorem, which states that a matrix satisfies its own characteristic equation,to compute the inverse of the 3×3 matrix:

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & -1 & -3 \\ -1 & 0 & 1 \end{pmatrix}.$$

(25) **Problem 25.**
Solve the non-homogeneous first order linear system of DE

$$x' = x - y + \cos(t), \;\; y' = y - x - e^{-t}$$

(26) **Problem 26.**
Give an example of a subset of the set of all upper triangular matrices which is closed under addition of matrices but it is not closed under multiplication by scalars.

(27) **Problem 27.**
Draw the phase portrait for the following DE:

$$y' = y^2(1 - y).$$

(28) **Problem 28.**
Solve the differential equation

$$(3x - 2y)y' = 3y$$

(29) **Problem 29.**
Let

$$y^2 = cx$$

be a one parameter family of curves in the plane. Find the orthogonal family of curves.

(30) **Problem 30**
Consider the population model

$$\frac{dP}{dt} = r(P - T)P, \;\; P(0) = P_0,$$

where r,T and P_0 are positive constants. What predictions can you make regarding the behavior of the population? Consider the cases $P_0 < T$ and $P_0 > T$.

(31) **Problem 31**
Let A be a 2×2 matrix having 1 and -1 as its eigenvalues. Such a matrix is not unique.Show that

$$A^2 = I_2.$$

(32) **Problem 32**
Discuss the existence and uniqueness of solutions to the IVP

$$y' = 3xy^{1/3}, \;\; y(0) = 0.$$