

Practice Gateway II for 1372

Take the practice gateway just as you would in an actual gateway:

* **30 minutes**

* No books, notes, or calculators allowed

These formulae will be given to you in the gateway:

$$2 \cos^2 x = 1 + \cos 2x$$

$$2 \sin^2 x = 1 - \cos 2x$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$1. \quad \int \frac{x^3 - 4x + 7x^{3/2}}{x^2} dx$$

$$2. \quad \int_0^1 x \sqrt[3]{3x^2 + 8} dx$$

$$3. \quad \int \cos^2 x \sin^3 x \, dx$$

$$4. \quad \int \sin^2(3x/2) \, dx$$

$$5. \int \frac{x+6}{(2-x)(3x+2)} dx$$

$$6. \int x \cos 2x dx$$

$$7. \quad \int \frac{3}{7 - 2x} dx$$

$$8. \quad \int \frac{3}{x^2 + 16} dx$$

9. Convert the following integral to an integral with variable θ using the substitution $2x = \sin \theta$. The values of the new limits must be in radians.

$$\int_{1/4}^{1/2} \frac{x^3}{\sqrt{1 - 4x^2}} dx$$

10. $\int x \sec^2 x dx$

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1.
$$\begin{aligned} \int \frac{x^3 - 4x + 7x^{3/2}}{x^2} dx &= \int x - 4x^{-1} + 7x^{-1/2} dx \\ &= \frac{x^2}{2} - 4 \ln|x| + 14x^{1/2} + C \end{aligned}$$
2.
$$\begin{aligned} \int_0^1 x \sqrt[3]{3x^2 + 8} dx &= \int_0^1 x(3x^2 + 8)^{1/3} dx = \frac{1}{6} \int_8^{11} u^{1/3} du \quad (u = 3x^2 + 8 \Rightarrow du = 6x dx) \\ &= \frac{1}{6} \left[\frac{u^{4/3}}{4/3} \right]_8^{11} = \frac{1}{8}(11^{4/3} - 8^{4/3}) = \frac{1}{8}(11)^{4/3} - 2 \end{aligned}$$
3.
$$\begin{aligned} \int \cos^2 x \sin^3 x dx &= \int \cos^2 x \sin^2 x \sin x dx = \int \cos^2 x (1 - \cos^2 x) \sin x dx \\ &= - \int u^2 (1 - u^2) du \quad (u = \cos x \Rightarrow du = -\sin x dx) \\ &= -\frac{u^3}{3} + \frac{u^5}{5} + C = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C \end{aligned}$$
4.
$$\int \sin^2(3x/2) dx = \int \frac{1 - \cos 3x}{2} dx = \int \frac{1}{2} - \frac{\cos 3x}{2} dx = \frac{x}{2} - \frac{\sin 3x}{6} + C$$
5.
$$\begin{aligned} \int \frac{x+6}{(2-x)(3x+2)} dx &= \int \frac{1}{2-x} + \frac{2}{3x+2} dx = - \int \frac{-1}{2-x} dx + \frac{2}{3} \int \frac{3}{3x+2} dx \\ &= -\ln|2-x| + \frac{2}{3} \ln|3x+2| + C \end{aligned}$$
6.
$$\begin{aligned} \int x \cos 2x dx &= \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} dx \quad (u = x \Rightarrow du = dx, dv = \cos 2x dx \Rightarrow v = \frac{\sin 2x}{2}) \\ &= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + C \end{aligned}$$
7.
$$\int \frac{3}{7-2x} dx = \frac{3}{-2} \int \frac{-2}{7-2x} dx = -\frac{3}{2} \ln|7-2x| + C$$
8.
$$\begin{aligned} \int \frac{3}{x^2 + 16} dx &= \int \frac{3}{16u^2 + 16} 4du \quad (x = 4u \Rightarrow dx = 4du) \\ &= \frac{3}{4} \int \frac{1}{u^2 + 1} du = \frac{3}{4} \tan^{-1} u + C = \frac{3}{4} \tan^{-1} \frac{x}{4} + C \end{aligned}$$
9.
$$\begin{aligned} \int_{1/4}^{1/2} \frac{x^3}{\sqrt{1-4x^2}} dx &= \int_{\pi/6}^{\pi/2} \frac{\frac{1}{8} \sin^3 \theta}{\sqrt{1-\sin^2 \theta}} \cdot \frac{\cos \theta}{2} d\theta \quad (2x = \sin \theta \Rightarrow 2dx = \cos \theta d\theta) \\ &= \frac{1}{16} \int_{\pi/6}^{\pi/2} \sin^3 \theta d\theta \end{aligned}$$
10.
$$\begin{aligned} \int x \sec^2 x dx &= x \tan x - \int \tan x dx \quad (u = x \Rightarrow du = dx, dv = \sec^2 x dx \Rightarrow v = \tan x) \\ &= x \tan x + \int \frac{-\sin x}{\cos x} dx = x \tan x + \ln|\cos x| + C \end{aligned}$$