Homework Problem Set #4

Willard Miller

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Exercise 1 *What is the frequency response for the maxflat Daubechies filter with $p = 2$? Graph $H(\omega)$. What are its symmetries?*

Exercise 2 *Compute $\mathbf{h}(n)$ for the half band Daubechies filter with $p = 5$. Verify that $H(e^{i\omega})$ has four zero derivatives at $\omega = 0$ and $\omega = \pi$.*

Exercise 3 *Find $H_1(z)$, $F_0(z)$ and $F_1(z)$ for the biorthogonal filter bank with*

$$P_0(z) = \frac{1}{16}\left(-1 + 9z^{-2} + 16z^{-3} + 9z^{-4} - z^{-6}\right), \qquad H_0(z) = \left(\frac{1+z^{-1}}{2}\right)^3.$$

Exercise 4 *Let $\phi(t)$ and $w(t)$ be the Haar scaling and wavelet functions. Let V_j and W_j be the spaces generated by $\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k)$ and $w_{j,k}(t) = 2^{j/2}w(2^j t - k)$, $k = 0, \pm 1, \cdots$, respectively. Let $f(t)$ be defined on $0 \le t < 1$ and given by*

$$f(t) = \begin{cases} -1 & 0 \le t < 1/4 \\ 4 & 1/4 \le t < 1/2 \\ 2 & 1/2 \le t < 3/4 \\ -3 & 3/4 \le t < 1. \end{cases}$$

1. *Express f in terms of the basis for V_2.*

2. *Decompose f into its component parts in W_1, W_0, and V_0. In other words, find the Haar wavelet decomposition for f.*

3. *Sketch each of the four decompositions.*

Exercise 5 *Suppose that* $\{V_j : j = 0, \pm 1, \cdots\}$ *is a multiresolution analysis with scaling function* $\phi(t)$ *and that* ϕ *is continuous and compactly supported.*

1. *Find* $\Pi_j(t)$*, the orthogonal projection onto* V_j *of the step function*

$$\Pi(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & t < 0 \text{ or } t > 1. \end{cases}$$

2. *If* $\int_{-\infty}^{\infty} \phi(x)dx = 0$*, show that for all* j *sufficiently large,* $||\Pi - \Pi_j|| \geq \frac{1}{2}$.

3. *Explain why the preceding result implies that* $\int_{-\infty}^{\infty} \phi(x)dx \neq 0$,