Homework Problem Set #7

Math 5467

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Exercise 1 *Problem 4 in Section 6.4 (page 208) of your text: If* $H(\omega)$ *has* p *zeros at* $\omega = \pi$ *show that* $\hat{\phi}(\omega)$ *has* p *zeros at* $\omega = 2\pi n$ *for each* $n \neq 0$.

Exercise 2 *Problem 4 in Section 6.5 (page 218) of your text: What wavelets come from the biorthogonal filters with* $H_0 = 1$, $F_0 = \frac{1}{2}z + 1 + \frac{1}{2}z^{-1}$, $H_1 = \frac{1}{2}z - 1 + \frac{1}{2}z^{-1}$,$F_1 = -1$*? Recognize the delta (see example 6.2 on page 184) and hat:*

$$\tilde{\phi}(t) = 2\tilde{\phi}(2t) \quad \text{and} \quad \phi(t) = \frac{1}{2}\phi(2t+1) + \phi(2t) + \frac{1}{2}\phi(2t-1).$$

Then construct wavelets from $\tilde{w}(t) = -\frac{1}{2}\tilde{\phi}(2t+1) + \tilde{\phi}(2t) - \frac{1}{2}\tilde{\phi}(2t-1)$ *and* $w(t) = 2\phi(2t-1)$. *Check the biorthogonality conditions*

$$\int \phi(t)\tilde{\phi}(t-k)dt = \int w(t)\tilde{w}(t-k)dt = \delta(k),$$

$$\int \phi(t)\tilde{w}(t-k)dt = \int \tilde{\phi}(t)w(t-k)dt = 0.$$

Exercise 3 *Verify that if* $g \in L^2(-\infty,\infty)$, $||g|| = 1$ *and* g *is centered about* (t_0, ω_0) *in phase space, then the windowed Fourier function* $g^{[x_1,x_2]}$ *is centered about* $(t_0 - x_1, \omega_0 + x_2)$.

Exercise 4 *Given the function*

$$g(t) = \begin{cases} 1, & |t| \leq \frac{1}{2} \\ 0, & |t| \geq \frac{1}{2}, \end{cases}$$

show that the set $\{g^{[m,n]}\}$ *is an ON basis for* $L^2(-\infty,\infty)$. *Here,* m, n *run over the integers.*

Exercise 5 *Suppose* $g \in L_2(-\infty, \infty)$ *with* $||g|| = 1$ *and* g *is centered about* $(0, k)$ *in the position-momentum space. Show that the continuous mother wavelet* $g^{(a,b)}$ *is centered about* $(b, a^{-1}k)$.