Homework Problem Set #4

Math 5467

March 22, 2004

Exercise 1 *Find the z-transform for the sequence* $\mathbf{x}(n) = 0$ *for* $n < 0$ *and* $\mathbf{x}(n) = (1/2)^n$ *for* $n \geq 0$*, i.e.,*

$$\mathbf{x} = \begin{pmatrix} \cdots & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{4} & \cdots & \frac{1}{2^n} & \cdots \end{pmatrix}$$

Exercise 2 *Show explicitly that the double-shift orthogonality condition in the frequency domain*

$$\int_{-\pi}^{\pi} e^{2ik\omega} |C(\omega)|^2 d\omega = 2\pi\delta_{k0}$$

for integer k*, is equivalent to*

$$|C(\omega)|^2 + |C(\omega + \pi)|^2 = 2.$$

Exercise 3 *Plot the frequency response function* $|H(\omega)|$ *in the interval* $0 \leq \omega \leq 2\pi$ *for the Daubechies 4 tap filter. Point out the features of the graph that cause this filter to belong to the maxflat class.*

Exercise 4 *Let* $\phi(t)$ *and* $w(t)$ *be the Haar scaling and wavelet functions. Let* V_j *and* W_j *be the spaces generated by* $\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k)$ *and* $w_{j,k}(t) = 2^{j/2} w(2^j t - k)$*,* $k = 0, \pm 1, \cdots$*, respectively. Let* $f(t)$ *be defined on* $0 \leq t < 1$ *and given by*

$$f(t) = \begin{cases} 4 & 0 \leq t < 1/4 \\ -1 & 1/4 \leq t < 1/2 \\ 0 & 1/2 \leq t < 3/4 \\ 3 & 3/4 \leq t < 1. \end{cases}$$

1. *Express* f *in terms of the basis for* V_2.

2. *Decompose* f *into its component parts in* W_1, W_0, *and* V_0. *In other words, find the Haar wavelet decomposition for* f.

3. *Sketch each of the four decompositions.*

Exercise 5 *Let* $\phi(t)$ *and* w *be the Haar scaling and wavelet functions, respectively. Let* V_j *and* W_j *be the spaces generated by* $\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k)$ *and* $w_{j,k}(t) = 2^{j/2}w(2^j t - k)$, $k = 0, \pm 1, \cdots$, *respectively. Suppose the real valued function* $f(t) = \sum_k a_k \phi_{1,k}(t)$ *belongs to* V_1 *and* $f \perp V_0$. *Show that* $a_{2\ell+1} = -a_{2\ell}$ *for all integers* ℓ. *Conclude that* f *can be expressed as a linear combination of the functions,* $w_{0,k}(t)$, *hence that* $f \in W_0$.