Homework Problem Set #5

Math 5467

April 30, 2004

Exercise 1 *This first problem is not to be turned in. It is an introduction to interactive demos and scripts that you can use to familiarize yourself with the wavelet capabilities of the MatLab wavelet toolbox. (If you do not have access to the wavelet toolbox, let me know and I will give you some general MatLab code that will that will implement the basic wavelet decomposition and reconstruction algorithms.)*

Only one user at a time can access the MatLab Wavelet Toolbox on the School of Mathematics linux machines. However MatLab itself is available to all users. Once you login on a Math Department computer type **matlab** *to open MatLab. Then type* **wavemenu** *to open the wavelet toolbox. If the wavelet toolbox is available the toolbox menu will open. The following interactive demos are directly relevant to this part of the course:*

1. *Wavelet 1-D. Press the* **Wavelet 1-D** *option. Under the File menu option, Demo Analysis you can select various sample signals for analysis.*

2. *Wavelet Display. Press the* **Wavelet Display** *option. This shows you the definitions, graphs and properties of some standard scaling functions.*

3. *Denoising in 1-D. Press the* **Wavelet 1-D** *option. Under the File menu option, Demo Analysis you can select various sample signals for denoising.*

4. *Discontinuity Detection. Press the* **Wavelet 1-D** *option. Under the File menu option, Demo Analysis you can select various sample signals for discontinuity detection.*

Exercise 2 *Reconstruct a signal $f \in V_3$ given that its only nonzero coefficients in the Haar wavelet decomposition are*

$$\{a_{2,k}\} = \{\frac{1}{2}, 2, \frac{5}{2}, -\frac{3}{2}\}, \qquad \{b_{2,k}\} = \{-\frac{3}{2}, -1, \frac{1}{2}, -\frac{1}{2}\}.$$

Here the first entry in each list corresponds to $k = 0$. Sketch the graph of f.

Exercise 3 *Reconstruct a signal $g \in V_3$ given that its only nonzero coefficients in the Haar wavelet decomposition are*

$$\{a_{1,k}\} = \{\frac{3}{2}, -1\}, \qquad \{b_{1,k}\} = \{-1, -\frac{3}{2}\}, \qquad \{b_{2,k}\} = \{-\frac{3}{2}, -\frac{3}{2}, -\frac{1}{2}, -\frac{1}{2}\}.$$

Here the first entry in each list corresponds to $k = 0$. Sketch the graph of g.

Exercise 4 *Let*

$$f(t) = \exp(-\frac{t^2}{10}) \left(\sin(2t) + 2\cos(4t) + 0.4\sin(t)\sin(50t)\right).$$

Discretize (sample) the function $f(t)$ over the interval $0 \le t \le 1$ as described in Exercise 4 of Homework Problem Set #3, so that the discretization belongs to V_8 for the Haar decomposition. In other words, use $J = 8$ as the top level so that there are $2^8 = 256$ nodes in the discretization. Implement the FWT decomposition algorithm for Haar wavelets, db1. Plot the resulting levels $f_j \in V_j$, $j = 0, \cdots, 7$ and compare with the original signal. Here you can use one of

1. *the display facilities of the MatLab wavelet toolbox. Note: You can prepare a data file with commands such as*

```
t=linspace(0,1,2^10)
```

and

```
f=exp(-t.^2/10).*(sin  (2*t)+2*cos(4*t)+0.4*sin(t).*sin(50*t))
```

The command

```
save  function f
```

will save the vector f in the data file $function.mat$. Then for the "load signal" option you simply open the file function.mat *and use the analysis and processing functions of the wavelet toolbox.*

2. *the MatLab Wavelet Toolbox commands* **wavedec** *and* **waverec**. **wavedec** *is the Matlab wavelet decomposition routine. Its inputs are the discretized signal vector y, the number of levels J (so that for a maximal decomposition number of nodes is 2^J = length of y = length of t), and 'dbp' where* **dbp** *is the appropriate Daubechies wavelet, i.e.,* **db1** *is the Haar wavelet,* **db2** *is the Daubechies 4-tap wavelet, etc. The appropriate command is*

   ```
   [C,L] = wavedec(y,J,'dbp')
   ```

   Recall that **y** *is associated to the time nodes t (e.g.,*

   ```
   t=linspace (0,1,2^8)
   ```

   to decompose the interval $[0, 1]$ into 2^8 terms). The output C is the vector consisting of the $a_{0,k}$ and $b_{0,k}, b_{1,k}, \cdots b_{J-1,k}$ coefficients, in that order. L is a bookkeeping parameter. The wavet recovery command, to obtain the signal **yr** *from the wavelet coefficients, is*

   ```
   yr = waverec(C,L,'dbp')
   ```

   . This is just the inverse of the **wavedec** *command. However, one can process the C output of* **wavedec** *and obtain a modified file of wavelet coefficients CM. Then the command*

   ```
   yr = waverec(CM,L,'dbp')
   ```

will give the reconstruction of the modified signal. A related useful comand is

```
[NC,NL,cA] = upwlev(C,L,'dbp')
```

a one-dimensional wavelet analysis function. This comand performs the single-level reconstruction of the wavelet decomposition structure $[C, L]$ *giving the new one* $[NC, NL]$*, and extracts the last approximation coefficients vector* cA*. Thus since* $[C, L]$ *is a decomposition at level* J*,* $[NC, NL]$ *is the same decomposition at level* $J - 1$ *and* cA *is the approximation coefficients vector at this level. The default signal extension for these algorithms is zero padding.*

3. *or I will give you you some general MatLab code that will implement the basic wavelet decomposition and reconstruction algorithms.*

Exercise 5 *Repeat Exercise 4 using* **db2***, the Daubechies 4-tap wavelets.*

Exercise 6 *Compression. Filter the wavelet coefficients computed in Exercise 4 by setting to zero any wavelet coefficient whose absolute value is less than tol* $= 0.1$*. (Here, one option is to use the command* **compress** *whose code was given in Homework Set #3.) Then reconstruct the signal using the Haar reconstruction algorithm. Plot the reconstructed* f_8 *and compare with the original signal. Compute the relative* L^2 *difference in norm between the original and compressed signals. Experiment with various tolerances. Keep track of the percentage of coefficients that have been set equal to zero.*

Exercise 7 *(Polynomial Suppression) Sometimes signals have a built-in bias, a polynomial part (say linear, quadratic or cubic) added to a bounded and rapidly oscillating part. We would like to remove the bias from the signal. Take the example*

$$f(t) = 2 + 4t + t^3 + \frac{1}{60}\sin(64\pi t)\cos(6\pi t).$$

Suppose we have 1024 samples of $f(t)$ *on the interval* $[-1, 1]$*.*

1. *Devise a strategy for separating the bias from the remaining signal, via Daubechies wavelet analysis. Hint: How big does* N *have to be to reproduce cubics exactly in* V_0*?*

2. *Use MatLab to carry out the decomposition. You will have to pay attention to the fact that the signal is finite. Thus some type of signal extension is needed at the data boundaries.*