

6628 Theorems on Infinite Series Spg 05

1. For a geometric series $a_k = a_1 r^{k-1}$ and $\sum_{k=1}^n a_1 r^{k-1} = \frac{a_1(1 - r^n)}{1 - r}$.
2. For an arithmetic series $a_k = a_1 + (k-1)d$ and $\sum_{k=1}^n a_k = \frac{n}{2}[a_1 + a_n]$.
3. If $\lim_{n \rightarrow \infty} a_n = L$ and $L \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is divergent.
4. Comparison test. If $0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
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6. The p series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.
7. If $0 < b_{n+1} \leq b_n$ and $\lim_{n \rightarrow \infty} b_n = 0$, then $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ is convergent.
8. If $\lim_{n \rightarrow \infty} b_n = L$ and $L \neq 0$, then $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ diverges.
9. If $0 < b_{k+1} \leq b_k$ and $\lim_{k \rightarrow \infty} b_k = 0$, then
$$|R_N| = \left| \sum_{k=N+1}^{\infty} (-1)^{k-1} b_k \right| \leq b_{N+1}.$$
10. Let $L = \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|}$. If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges. If $L > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.
11. The general formula for the Taylor's Series for $f(x)$ at $a = 0$ is
$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$