Math 5467. Midterm Exam II (take-home problem)

April 12, 2004

Let $\phi(t)$ be a continuous scaling function satisfying the dilation equation

$$\phi(t) = \sqrt{2} \sum_{\ell=0}^{N} \mathbf{c}(\ell) \phi(2t - \ell)$$

where the filter coefficients are normalized by

$$\sum_{k=0}^{N} \mathbf{c}(k) = \sqrt{2}$$

and $\phi(t)$ is normalized by $\int \phi(t)dt = 1$. In class and in the notes, we show that $\sum_k \phi(t+k) = 1$ for all t, so in particular $\sum_k \phi(k) = 1$. From these facts alone, show that

$$\sum_k \phi(\frac{k}{2^j}) = 2^j \qquad j = 0, 1, 2, \cdots.$$

Hint: Try induction on j.