Math 1571H, Fall 2005
Solution to Quiz 2 (September 22)

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1) [4 points] Since

$$\lim_{x\to 4} \frac{\sqrt{x^2+9}-5}{x^2-4x} = \frac{0}{0},$$

we should cancel $(x-4)$ from both numerator and denominator.

$$\lim_{x\to 4} \frac{\sqrt{x^2+9}-5}{x^2-4x} = \lim_{x\to 4} \frac{\sqrt{x^2+9}-5}{x(x-4)} . \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5} = \lim_{x\to 4} \frac{(x-4)(x+4)}{x(x-4)(\sqrt{x^2+9}+5)} = \lim_{x\to 4} \frac{x+4}{x(\sqrt{x^2+9}+5)} = \frac{2}{10}.$$

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2)[3 points] $\lim_{x\to 0} \frac{3\sin(x)}{\sqrt{x}} = 3\lim_{x\to 0} \frac{\sin(x)}{\sqrt{x}} . \frac{\sqrt{x}}{\sqrt{x}} = 3. \lim_{x\to 0} \sqrt{x}. \lim_{x\to 0} \frac{\sin(x)}{x} = 3.0.1 = 0.$

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3) [3 points] Let $P = (0, 0, z)$ be a point on the z-axis which is equidistant from $A = (2, 5, -3)$ and $B = (-3, 6, 1)$. That is, $|\overrightarrow{PA}| = |\overrightarrow{PB}|$. Therefore,

$$2^2 + 5^2 + (z+3)^2 = 3^2 + 6^2 + (z-1)^2.$$

Simplify, $z = 1$.

[2 points] Bonus: Determine whether the following is True or False. Explain.

1) If $\lim_{x\to 6} f(x)g(x)$ exists, then the limit must be $f(6)g(6)$.
FALSE. As a counter-example, let $f(x) = (x-6)$ and $g(x) = 1/(x-6)$. We have $\lim_{x\to 6} f(x)g(x) = 1$; but $f(6) = 0$, and $g(6)$ is unbounded, so $f(6)g(6)$ is undetermined.

2) If $p(x)$ is a polynomial, then $\lim_{x\to b} p(x) = p(b)$.
TRUE. Since all polynomials are continuous.