Math 1571H, Fall 2005
Solution to Quiz 3 (September 29)

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1) [5 points] Find the distance between the lines

$$\ell_1: \frac{x-1}{-1} = \frac{y-3}{4} = \frac{z}{2} \quad \text{and} \quad \ell_2: \frac{x+1}{1} = \frac{y+1}{2} = \frac{z-2}{0}.$$

Solution: Let $\overrightarrow{u}_1, \overrightarrow{u}_2$ be the directions of ℓ_1, and ℓ_2 respectively. $\overrightarrow{u}_1 = -i + 4j + 2k$, and $\overrightarrow{u}_2 = i + 2j$. Take any point P on ℓ_1, say $P = (1,3,0)$, and any point, say $Q = (-1,-1,2)$, on ℓ_2. Find the direction of the normal, $\overrightarrow{n}$, to both ℓ_1 and ℓ_2 by calculating $\overrightarrow{n} = \overrightarrow{u}_1 \times \overrightarrow{u}_2 = -4i + 2j - 6k$. The distance between the two lines, d, is calculated as follows:

$$d = \left| \overrightarrow{PQ} \cdot \frac{\overrightarrow{n}}{|\overrightarrow{n}|} \right| = \left| (-2i - 4j + 2k) \cdot \frac{-4i + 2j - 6k}{\sqrt{56}} \right| = \frac{12}{\sqrt{56}}.$$

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2) [5 points] Find the equation of the *normal* line to the given curve at the given point.

$$y = \frac{x}{(3-x^2)^5}, \ (2,-2).$$

Solution: First, make sure that the point is on the curve by direct substitution. Then, find the slope of the tangent by calculating dy/dx at $(2,-2)$:

$$\frac{dy}{dx} = \frac{1}{(3-x^2)^5} + \frac{10x^2}{(3-x^2)^6}.$$

Therefore, the slope of the tangent at the given point is $m = -1 + 40 = 39$. So, the slope of the normal is $-1/39$. The equation of the normal is

$$(y+2) = -\frac{1}{39}(x-2).$$

[2 points] Bonus: Determine whether the following is True or False. Explain.

1) If f has an absolute minimum value at c, then $f'(c) = 0$.
FALSE. As a counter-example, consider $f(x) = |x|$ on either $[-c, c]$, $c > 0$ or the whole real line. The absolute minimum is at $x = 0$, but $f'(0)$ does not exist.

2) If f is differentiable, and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.
TRUE. Mean Value Theorem is a testimony for this.