Math 1571H, Fall 2005
Solution to Quiz 4 (October 13)

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1) [12 points] Give a **graceful labeled sketch** of the graph of

$$y = \frac{1}{x-1} - x.$$

You need to justify every step (i.e., Domain, asymptotes, $x-$intercept, $y-$intercept, symmetry, inflection points/concavity, local extremum, intervals of increase/decrease, etc.). Also, find the limits as $x \to 1^+$, and $x \to 1^-$.

Solution:

1. $D = \{\text{All } x \text{ in } \mathbb{R} \text{ such that } x \neq 1\}$
2. If $y = 0$, then $x = \frac{1}{x-1}$. That is, $x-$intercepts are $x = \frac{1\pm\sqrt{5}}{2}$. If $x = 0$, then $y-$intercept is $y = -1$.
3. No symmetry, since $f(-x) = -\frac{1}{x+1} + x$, which is equal to neither $f(x)$ nor $-f(x)$.
4. Asymptotes (i.e., guiding lines): There exist two asymptotes: a vertical asymptote at $x = 1$. $\lim_{x\to 1^+} y = \infty - 1 = \infty$, and $\lim_{x\to 1^-} y = -\infty - 1 = -\infty$. For very large x, $-x$ will be dominant over$\frac{1}{x-1}$, since $\lim_{x\to\pm\infty} \frac{1}{x-1} = 0$, and $\lim_{x\to\pm\infty} -x = \mp\infty$. So, there is a slant asymptote $y = -x$.
5. $f'(x) = -1 - 1/(x-1)^2 < 0$, for all $x \neq 1$, so f is decreasing on $(-\infty, 1)$ and $(1, \infty)$.
6. No solution to $f'(x) = 0$, and the only critical point is where $f'(x)$ does not exist at $x = 1$, which is not in D. No local extremum.
7. There is no solution to $f''(x) = 0$, and $f''(x)$ does not exist at $x = 1.$, which is not in D. No inflection points.
8. $f''(x) = \frac{2}{(x-1)^3} > 0 \Leftrightarrow x > 1$, so f is concave up on $(1, \infty)$; and f is concave down on $(-\infty, 1)$, since $f'' < 0$ on $(-\infty, 1)$.

The sketch will be given on the board next class meeting. Also, use your graphing calculator or any software to plot it.