Math 1571H, Fall 2005
Solution to Quiz 5 (October 27)

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1) [3 points] Calculate the integral

$$\int \frac{(2x+3)^2}{\sqrt[3]{x}}dx = \int \frac{4x^2+12x+9}{\sqrt[3]{x}}dx = \int 4x^{5/3}+12x^{2/3}+9x^{-1/3}dx = \frac{3x^{8/3}}{2}+\frac{36x^{5/3}}{5}+\frac{27x^{2/3}}{2}+C.$$

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2) [3 points] Compute the integrals by using the given substitution

$$I = \int x^{2/3}(2-x^{5/3})^{-4}dx, \quad u = 2-x^{5/3}.$$

Since $du = -\frac{5}{3}x^{2/3}dx$, we get

$$I = \int -\frac{3}{5}u^{-4}du = \frac{1}{5}u^{-3}+C = \frac{1}{5}(2-x^{5/3})^{-3}+C.$$

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3) [4 points] Find the particular solution of the following differential equation that satisfies the given initial condition:

$$\frac{dy}{dt} = \frac{4+4t^3}{2+2y}, \quad y=2, \text{ when } t=0.$$

Use separation of variables, you get

$$\int (2+2y)dy = \int (4+4t^3)dt.$$

Then,

$$2y+y^2 = 4t+t^4+C.$$

Use Initial Conditions,

$$2(2)+(2)^2 = 0+C, \quad \text{that is } C=8.$$

Thus, the particular solution is

$$2y+y^2 = 4t+t^4+8.$$

[1 point Bonus] Determine whether the following statement is True or False with justification.
The most general form of the antiderivative of $f(x)=\frac{1}{x^2}$ over its domain is $F(x)=-\frac{1}{x}+C$, for some constant C in $\mathbb{R}$.

FALSE, The general form should be $F(x)=-\frac{1}{x}+C_1$ on $(-\infty,0)$, and$F(x)=-\frac{1}{x}+C_2$ on $(0,\infty)$, where C_1, C_2 in $\mathbb{R}$.