1. Find the suprema, infima, maxima and minima of the given subsets of \( \mathbb{R} \). If any of those do not exist, say so.

(a) \[ S = \left\{ \frac{n + 2}{n} : n \in \mathbb{N} \right\} \]

(b) \[ S = \left\{ \frac{2n - 1}{n} : n \in \mathbb{N} \right\} \]

(c) \[ S = (2, 3) \]

(d) \[ S = [2, 3) \]

(e) \[ S = (2, 3] \cap \mathbb{Q} \]

(f) \[ S = \{ x : x \in (1, 2] \cap \mathbb{Q}, x^2 < 2 \} \]

2. Prove that if \( x \) and \( y \) are rational numbers, then so are \( x + y \) and \( xy \). (Hint: Just use the definition of a rational number).

3. Prove that if \( x \) is irrational and \( y \) is rational, then \( x + y \) is irrational. (Hint: Prove by contradiction).

4. Is it always true that if \( x \) is irrational and \( y \) is rational, then \( xy \) is irrational?

5. Prove that the irrational numbers are dense in \( \mathbb{R} \), that is, given any two real numbers \( x \) and \( y \) with \( x < y \), there exists an irrational number \( z \) such that \( x < z < y \). (Hint: Use Problem 4 along with the rational density theorem).