1. Consider the sequences, \( \{a_n\}_{n=1}^{\infty} \) and \( \{b_n\}_{n=1}^{\infty} \) whose \( n^{th} \) terms are given by \( a_n = 4 + \frac{1}{n} \) and \( b_n = 3 - \frac{2}{n} \).

(a) Let \( \epsilon = 0.1 \). Find positive integers \( N_1 \) and \( N_2 \) such that \( |a_n - 4| < \epsilon \) for all \( n \geq N_1 \) and \( |b_n - 3| < \epsilon \) for all \( n \geq N_2 \).

(b) Now improve your answers in part (a) by finding the smallest such \( N_1 \) and \( N_2 \) respectively. Is there any reason for these (smallest) \( N_1 \) and \( N_2 \) to be equal? Could you think of two different sequences for which these (smallest) \( N_1 \) and \( N_2 \) coincide?

(c) Now let \( \epsilon = 0.01 \). Repeat parts (a) and (b) for this \( \epsilon \). Observe how the smallest \( N_1 \) and \( N_2 \) have grown in size for this new value of \( \epsilon \).

(d) Can you repeat part (a) for any given positive \( \epsilon \), no matter how small it is? What can you conclude about the convergence of the sequences \( \{a_n\}_{n=1}^{\infty} \) and \( \{b_n\}_{n=1}^{\infty} \)?

2. Determine whether the given subsets of \( \mathbb{R} \) are neighbourhoods of 2.

(a) \( A = [1, 3] \)

(b) \( B = (1.999, 2.001) \)

(c) \( C = [2, 3] \)

(d) \( D = (1, 3) \cap \mathbb{Q} \)

(e) \( E = (-\infty, 0) \cup (1, 3) \cup \{\pi, 46\} \)

3. Is the sequence \( \left\{ \frac{3n + 7}{n} \right\}_{n=1}^{\infty} \) bounded? If so find upper and lower bounds.

4. Let \( \{a_n\}_{n=1}^{\infty} \) be a sequence. Prove that if the sequence converges to a real number, \( A \), then it is bounded. Is the converse of this statement true?