1. Let \( f : [\alpha, \beta] \to \mathbb{R} \) be increasing. Let
\[
L(x) = \sup \{ f(y) : y < x \} \quad \text{and} \quad U(x) = \inf \{ f(y) : y > x \}
\]
for each \( x \in (\alpha, \beta) \).
Let \( x_0 \in (\alpha, \beta) \). Prove that
\[
L(x_0) \leq f(x_0) \leq U(x_0).
\]
Hint: Let \( A = \{ f(y) : y < x_0 \} \) and let \( B = \{ f(y) : y > x_0 \} \) so that \( L(x_0) = \sup A \) and \( U(x_0) = \inf B \). You may prove the statement in two ways:

(i) Show that \( f(x_0) \) is an upper bound for \( A \) and a lower bound for \( B \).

(ii) Prove by contradiction. Assume \( f(x_0) < L(x_0) \). Using the fact that \( L(x_0) \) is the supremum of a certain set, derive a contradiction (A standard argument leads to contradicting the fact that \( f \) is increasing). Similarly, assume \( f(x_0) > U(x_0) \) and then derive a contradiction using the fact that \( U(x_0) \) is the infimum of \( B \).

2. Let \( E \subseteq \mathbb{R} \) and let \( f : E \to \mathbb{R} \) and let \( x_0 \in E \). Suppose that \( x_0 \) is NOT an accumulation point of \( E \). Prove that \( f \) is continuous at \( x_0 \).

3. Let \( f : \mathbb{N} \to \mathbb{R} \) be a function. By the previous problem, deduce that \( f \) is continuous.