MATH 1272: **Calculus II**

Discussion Instructor: Jodin Morey   moreyjc@umn.edu
Discussion Session Website: math.umn.edu/~moreyjc

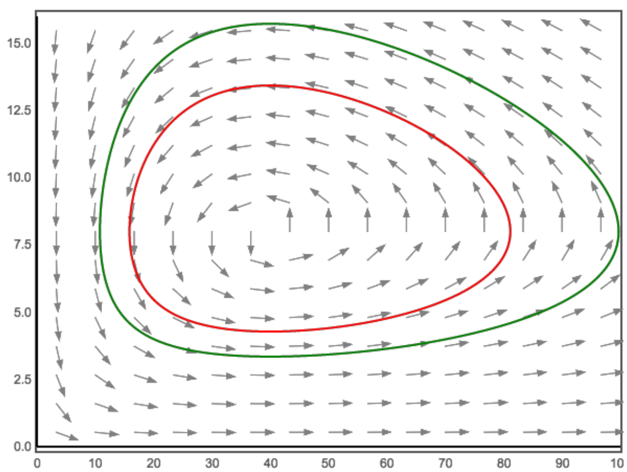
9.6 Predator-Prey Systems

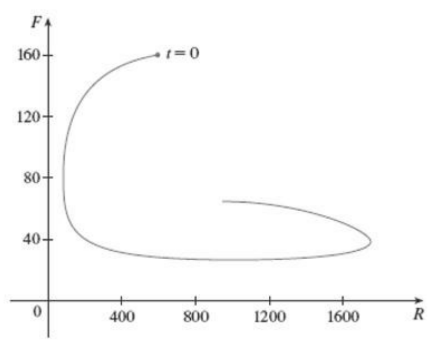
Review:

Predator-Prey Models: $W :=$ Wolves, $R :=$ Rabbits.
Without wolves, rabbits might have an exponential growth rate as we saw in a previous model: $\frac{dR}{dt} = k_1 R$.
Similarly, without rabbits, wolves would see an exponential decrease in their population: $\frac{dW}{dt} = -k_2 W$.
However, placed in the same environment, their populations can be modeled with: $\frac{dR}{dt} = k_1 R - aRW$ and $\frac{dW}{dt} = -k_2 W + bRW$, where $a, b > 0$.
The terms with RW allow us to measure the number of “interactions" (killings) that are likely to occur.

In order to discover if there are any population levels which would result in a stable system, we can set the rates of change, $\frac{dR}{dt}$ and $\frac{dW}{dt}$ equal to zero, and see if there are solutions to these equations. If so, these are called **Equilibrium Solutions**.

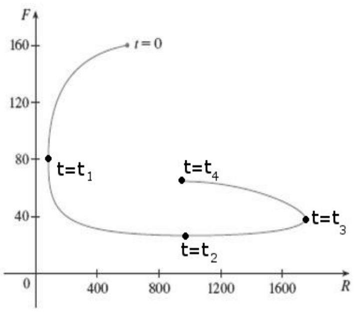
Phase plane





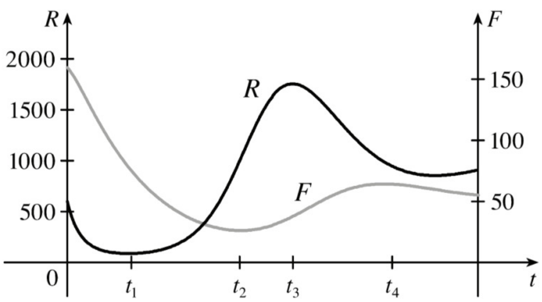
Problem #6   A phase trajectory is shown for populations of rabbits (R) and foxes (F).
a) Describe how each population changes as time goes by.

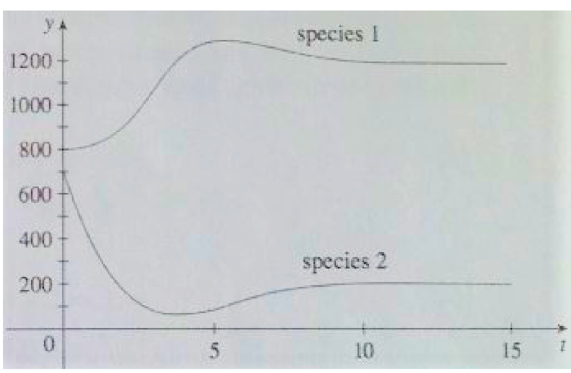
4.



At $t = 0$, there are about 600 rabbits and 160 foxes. At $t = t_1$, the number of rabbits reaches a minimum of about 80 and the number of foxes is also at 80. At $t = t_2$, the number of foxes reaches a minimum of about 25 while the number of rabbits has rebounded to 1,000. At $t = t_3$, the number of foxes has now increased to 40 and the rabbits population has reached a maximum of about 1,750. The curve ends at $t = t_4$, where the number of foxes has increased to 65 and the number of rabbits has decreased to about 950.

b) Use your description to make a rough sketch of the graphs of *R* and *F* as functions of time.





Problem #8 Graphs of populations of two species are shown. Use them to sketch the corresponding phase trajectory.

Problem #10 Populations of aphids and ladybugs are modeled by the equations:

$$\frac{dA}{dt} = 2A - 0.01AL,$$
$$\frac{dL}{dt} = -0.5L + 0.0001AL.$$

a) Find the equilibrium solutions and explain their significance.

A and L are constant $\Rightarrow$ $A' = 0$ and $L' = 0$

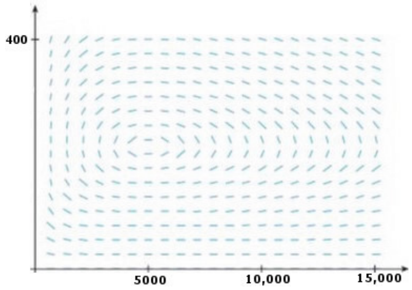
$$\Rightarrow \left\{ \begin{array}{c} 0 = 2A - 0.01AL, \\ 0 = -0.5L + 0.0001AL, \end{array} \right\} \Rightarrow \left\{ \begin{array}{c} 0 = A(2 - 0.01L), \\ 0 = L(-0.5 + 0.0001A). \end{array} \right.$$

So either $A = L = 0$, or $L = \frac{2}{0.01} = 200$ and $A = \frac{0.5}{0.0001} = 5000$.

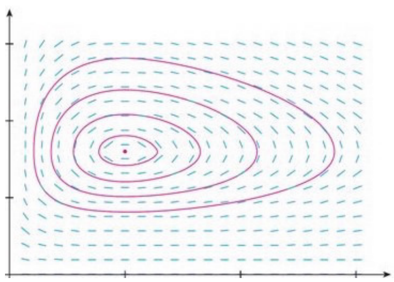
The trivial solution $A = L = 0$ just says that if there aren't any aphids or ladybugs,then the populations will not change. The nontrivial solution, $L = 200$ and $A = 5000$, indicates the population sizes needed so that there are no changes in either the number of aphids or the number of ladybugs.

b) Find an expression for $\frac{dL}{dA}$.

$$\frac{dL}{dA} = \frac{\frac{dL}{dt}}{\frac{dA}{dt}} = \frac{-0.5L+0.0001AL}{2A-0.01AL}.$$



c) The direction field for the differential equation in part b is shown. Use it to sketch a phase portrait. What do the phase trajectories have in common?



The solution curves (phase trajectories) are all closed curves that have the equilibrium point $(5000, 200)$ inside them.