**MATH 1271: Calculus I**

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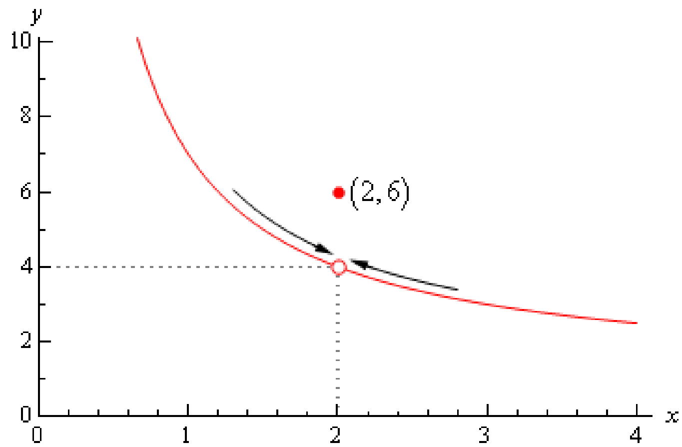
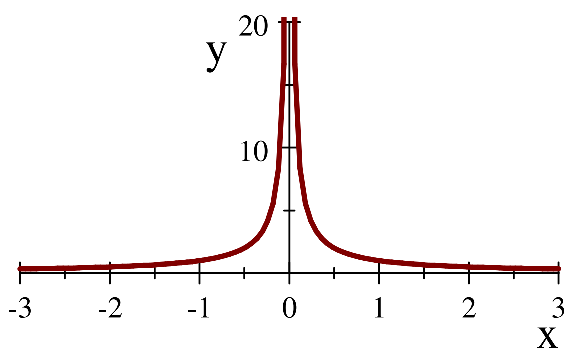
## 2.2 - Limit of a Function

### Review

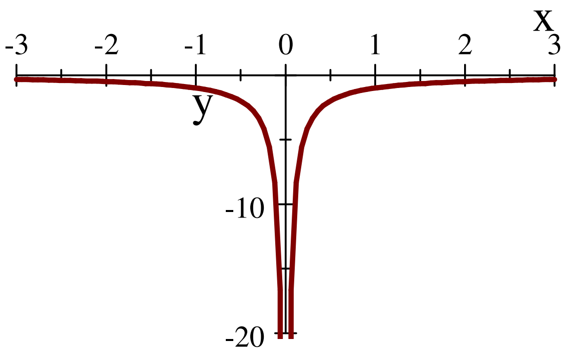



**Limit**: Suppose $f(x)$ is defined when $x$ is **near** the number $a$. This means that $f$ is defined on some open interval $(x_0, x_1)$ that contains $a$, except possibly not defined at $a$ itself. Then, if we can make the values of $f(x)$ **arbitrarily close to some number** $y = L$ (as close to $L$ as we like) by taking $x$ to be sufficiently close to $a$ (on either side of $a$) but not equal to $a$, then we write: $\lim_{x \to a} f(x) = L$, and say "the **limit** of $f(x)$, as $x$ approaches $a$, equals $L$."

**Positive Infinite Limit**: Let $f$ be a function defined on both sides of $a$, except possibly at $a$ itself. Then, we define $\lim_{x \to a} f(x) = \infty$ to mean that the value of $f(x)$ can be made arbitrarily large (as large as we please) by taking $x$ sufficiently close to $a$, but not equal to $a$.

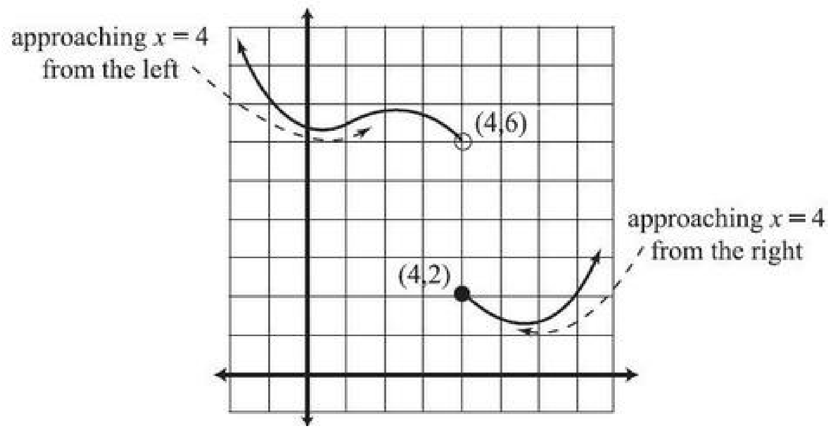



$\frac{1}{|x|}$



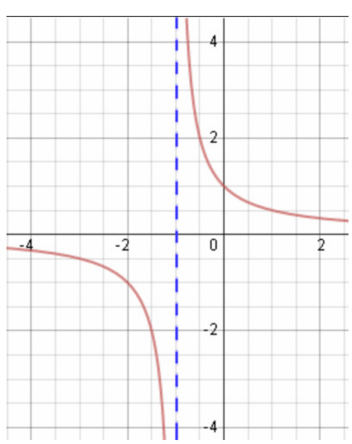

$-\frac{1}{|x|}$

**Negative Infinite Limit**: Let $f$ be defined on both sides of $a$, except possibly at $a$ itself. Then, we define $\lim_{x\to a} f(x) = -\infty$ to mean that the values of $f(x)$ can be made an arbitrarily large negative number by taking $x$ sufficiently close to $a$, but not equal to $a$.



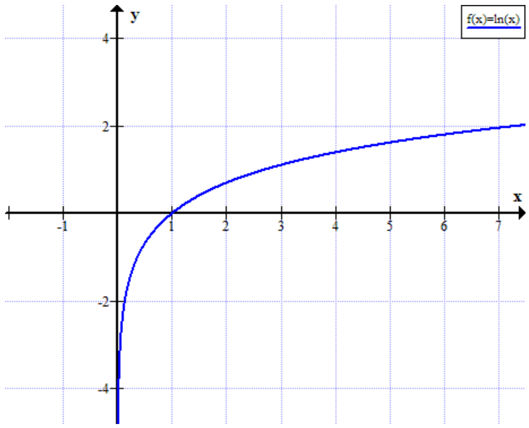

L.H.L. = 6,   R.H.L. = 2

**Left Hand Limit**: Let $f$ be defined on the left side of $a$, except possibly at $a$ itself. Then, we define $\lim_{x\to a^-} f(x) = L$ to mean that the values of $f(x)$ can be made an arbitrarily close to $L$ by taking $x$ sufficiently close to $a$ (from the left), but not equal to $a$.
A right-hand limit $\lim_{x\to a^+}$ is defined similarly.

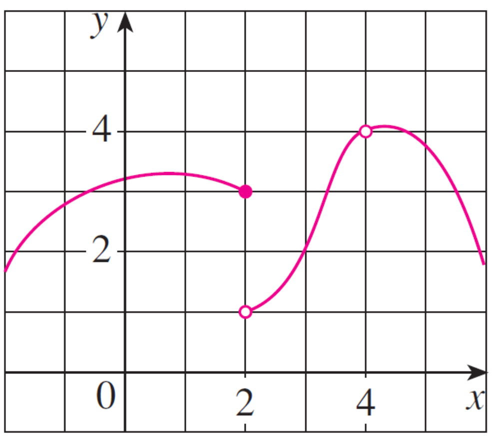



**Vertical Asymptote**: The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following statements is true: $\lim_{x\to a} f(x) = \pm\infty$, $\lim_{x\to a^-} f(x) = \pm\infty$, $\lim_{x\to a^+} f(x) = \pm\infty$.

Recall that: $\lim_{x \to 0^+} \ln x = -\infty$.




**Problem 4.** Use the given graph of $f$ to state the value of each quantity, if it exists. If it does not exist, explain why.



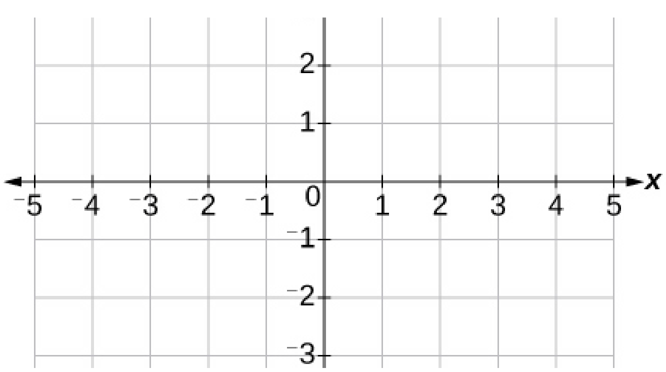

$\lim_{x \to 2^-} f(x)$ $\lim_{x \to 2^+} f(x)$

$\lim_{x \to 2} f(x)$ $f(2)$

$\lim_{x \to 4} f(x)$ $f(4)$

Problem 16. Sketch the graph of an example function $f$ that satisfies the conditions:

$\lim_{x \to 0} f(x) = 1,$ $\lim_{x \to 3^-} f(x) = -2,$ $\lim_{x \to 3^+} f(x) = 2,$ $f(0) = -1,$ $f(3) = 1$



**Problem** 33. Determine the infinite limit: $\lim_{x \to 3^+} \ln(x^2 - 9)$.

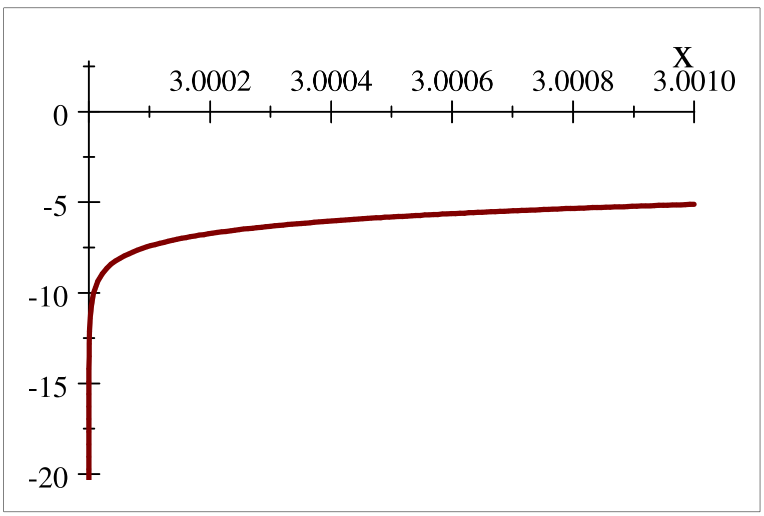
Recall we know that: $\lim_{x \to 0^+} \ln x = -\infty$.

So, let's use the substitution: $t = x^2 - 9$.

Then as $x \to 3^+$, $t \to 0^+$.

("Then as $x$ approaches 3 from the right, $t$ approaches 0 from the right")

So, we can make the substitutions into $\lim_{x \to 3^+} \ln(x^2 - 9)$, to get $\lim_{t \to 0^+} \ln t = -\infty$.




$\ln(x^2 - 9)$

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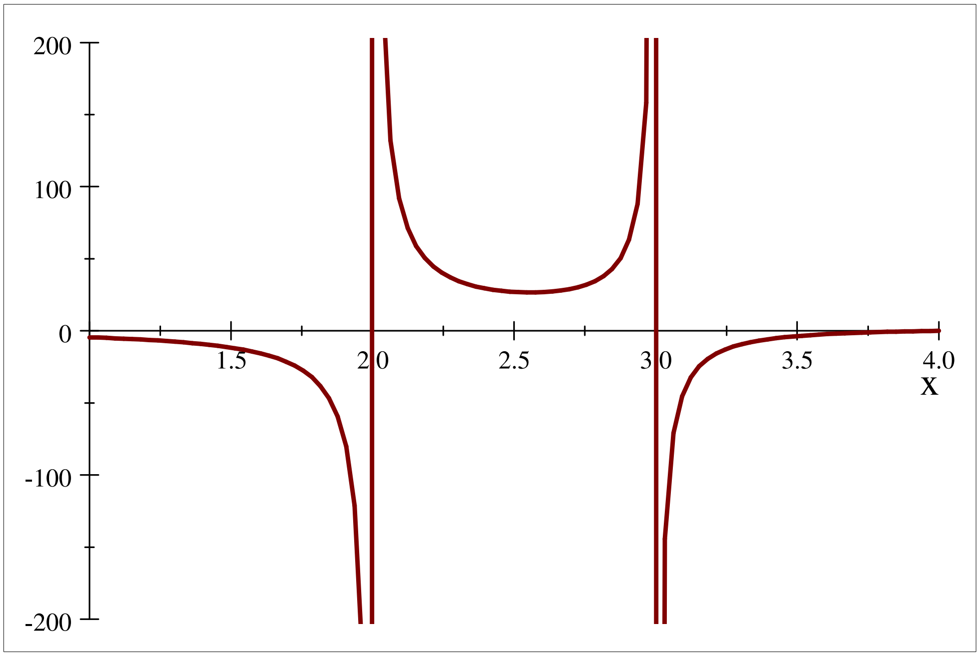
**Problem** 37. Determine the infinite limit: $\lim_{x \to 2^+} \frac{x^2-2x-8}{x^2-5x+6}$

$$= \lim_{x \to 2^+} \frac{(x-4)(x+2)}{(x-3)(x-2)} \to \frac{(-2^+)(4^+)}{(-1^+)0^+} = +\infty.$$

We get the last equality since the numerator is negative, and the denominator approaches 0 through negative values and $x \to 2^+$.




$$\frac{x^2-2x-8}{x^2-5x+6}$$