MATH 1271: Calculus I

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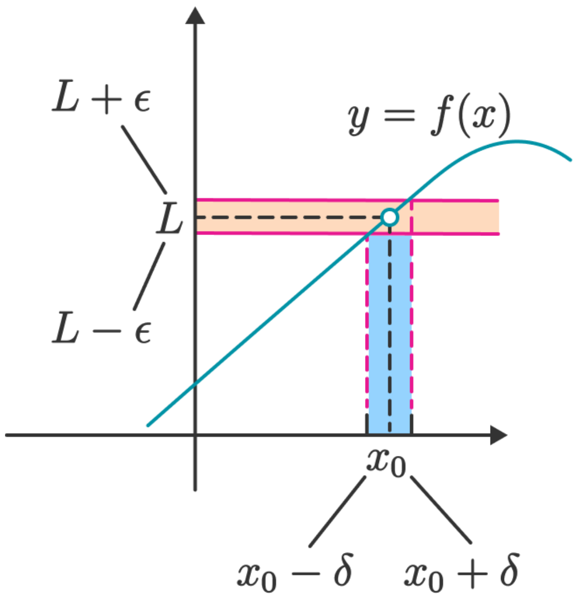
2.4 - Precise Definition of a Limit

Review:



Limit: We say that $\lim_{x\to a} f(x) = L$ if, for every number $\varepsilon > 0$, there exists a number $\delta > 0$ such that; if $(0 < |x-a| < \delta)$ then $(|f(x) - L| < \varepsilon)$.

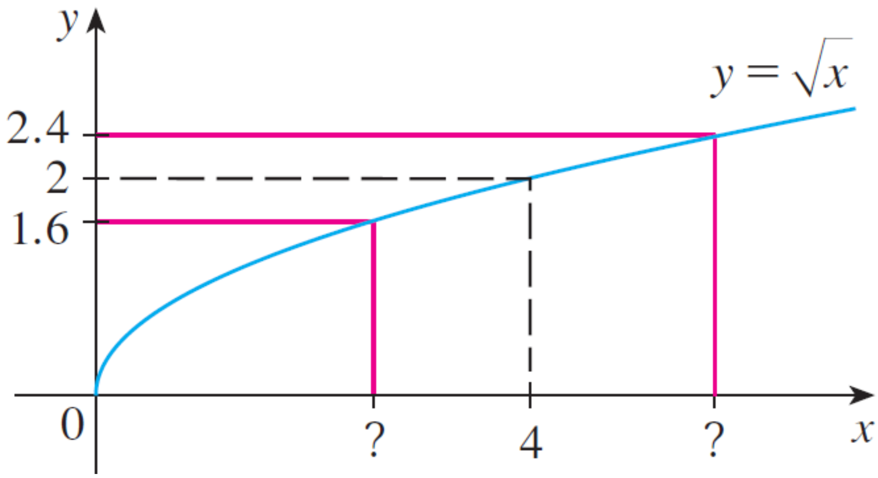
Left-Handed Limit: We say that $\lim_{x\to a^-} f(x) = L$ if, for every number $\varepsilon > 0$, there exists a number $\delta > 0$ such that; if $(a-\delta < x < a)$, then $(|f(x) - L| < \varepsilon)$.

Right-Handed Limit:
We say that $\lim_{x\to a^+} f(x) = L$ if, for every number $\varepsilon > 0$, there exists a number $\delta > 0$ such that; if $(a < x < a+\delta)$ then $(|f(x) - L| < \varepsilon)$.

Infinite Limits:
We say that $\lim_{x\to a} f(x) = \infty$ if, for every **positive** number M, there exists a **positive** number $\delta > 0$ such that; if $(0 < |x-a| < \delta)$ then $(f(x) > M)$.

Similarly, We say that $\lim_{x\to a} f(x) = -\infty$ if, for every **negative** number N, there exists a **positive** number $\delta > 0$ such that; if $(0 < |x-a| < \delta)$ then $(f(x) < N)$.

Problem 3.
Use the given graph of $f(x) = \sqrt{x}$ to find a number δ such that if $|x-4| < \delta$ then $|\sqrt{x} - 2| < 0.4$

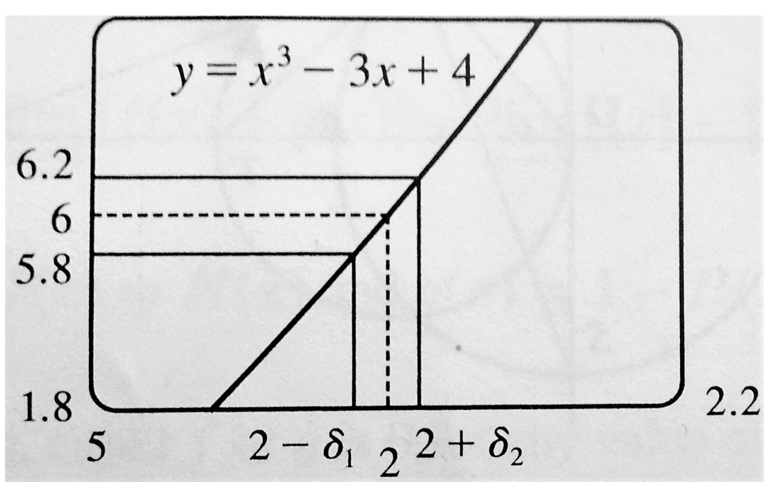


$1.6^2 = 2.56$
$2.4^2 = 5.76$

$|x-4| < |2.56-4| = 1.44$
$|x-4| < |5.76-4| = 1.76$

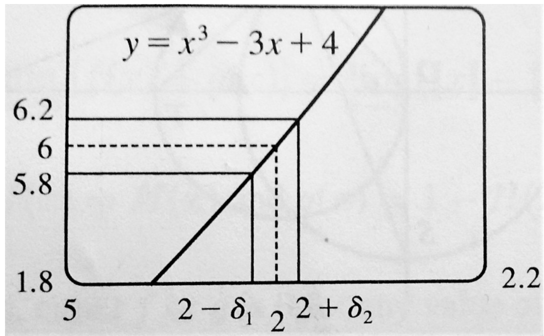
$\delta < 1.44$

Problem 7.
For the limit, $\lim_{x\to 2}(x^3-3x+4) = 6,$ illustrate the following definition, by finding values of δ that correspond to $\varepsilon = 0.2$.
Definition 2 (**from the book**): $\lim_{x\to 2} f(x) = 6$ if, for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that, if $(0 < |x-2| < \delta)$ then $(|f(x)-6| < \varepsilon)$.



For what x-value does the function output 5.8?
$y = x^3 - 3x + 4 = 5.8$ when $x \approx 1.9774,$

so $2 - \delta_1 \approx 1.9774 \Rightarrow \delta_1 \approx 0.0226$.



Also, $y = x^3 - 3x + 4 = 6.2$ when $x \approx 2.022$, so $2 + \delta_2 \approx 2.0219 \Rightarrow \delta_2 \approx 0.0219$.

Choosing the smaller of the two numbers, we get $\delta = 0.0219$.