# MATH 1271: Calculus I

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## 3.1 - Derivatives of Polynomials and Exponential Functions

### Review

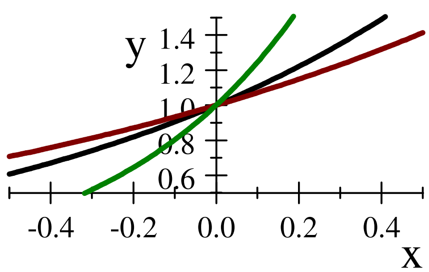
Derivatives of...

- **Constant Functions**: If $y = c$, where $c$ is some constant, then $y' = 0$.

- **Power Functions**: If $f(x) = x^r$, where $r$ is a real number, then $\frac{d}{dx}(x^r) = rx^{r-1}$.
  This includes if $f(x) = x$, where $\frac{d}{dx}(x^1) = 1 \cdot x^0 = 1$.

- **Constant Multiple**: If $c$ is a constant and $f$ is a differentiable function, then $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$.

- **Sum/Difference**: If $f$ and $g$ are both differentiable, then $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$.




$2^x$, $2.718^x$, and $3^x$

- **Euler's Number** $e$ : Imagine the family of functions: $f(x) = a^x$, where $a$ is any real number. $e$ is defined to be the number such that for $f(x) = a^x$, we have:
  $f'(0) = \lim_{h\to 0} \frac{a^{0+h} - a^0}{h} = \lim_{h\to 0} \frac{a^h - 1}{h} = 1$. (The slope is 1 at $x = 0$. The number turns out to be $e \approx 2.718$).

- **Exponential Functions**: If $f(x) = e^x$, then $f'(x) = e^x$. More generally, if $f(x) = a^x$, then $f'(x) = a^x \ln(a)$.

- **Polynomials**: Using Constant Multiple Rule and Sum/Difference Rule
  Say $y = ax^2 + bx^3 - c$, where $a, b, c$ are some constants. Then:

$$
\begin{aligned}
y' &= \tfrac{d}{dx}(ax^2) + \tfrac{d}{dx}(bx^3) - \tfrac{d}{dx}(c) \\
&= a\tfrac{d}{dx}(x^2) + b\tfrac{d}{dx}(x^3) - 0 \\
&= 2ax + 3bx^2.
\end{aligned}
$$

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**Random Problem**. Find the derivative of: $f(x) = 7x^2 - 7x^3 + 7e^x$

$$\frac{d}{dx}f(x) = \frac{d}{dx}[7x^2 - 7x^3 + 7e^x]$$

$$= 7\frac{d}{dx}[x^2 - x^3 + e^x]$$

$$= 7\left[\frac{d}{dx}(x^2) - \frac{d}{dx}(x^3) + \frac{d}{dx}(e^x)\right]$$

$$= 7[2x - 3x^2 + e^x]$$

$$= 14x - 21x^2 + 7e^x$$

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**Random Problem**. Find the derivative of: $f(t) = 1.7t^{2.3} - \frac{4}{\sqrt[3]{t}}$

$$f' = \frac{d}{dt}(1.7t^{2.3}) - \frac{d}{dt}\left(\frac{4}{\sqrt[3]{t}}\right)$$

$$f' = 1.7\frac{d}{dt}(t^{2.3}) - 4\frac{d}{dt}\left(t^{-\frac{1}{3}}\right)$$

$$f' = 1.7(2.3)t^{1.3} - 4(-\tfrac{1}{3})t^{-\frac{4}{3}}$$

$$f' = 3.91t^{1.3} + \tfrac{4}{3}t^{-\frac{4}{3}}.$$

**Second Derivative**…

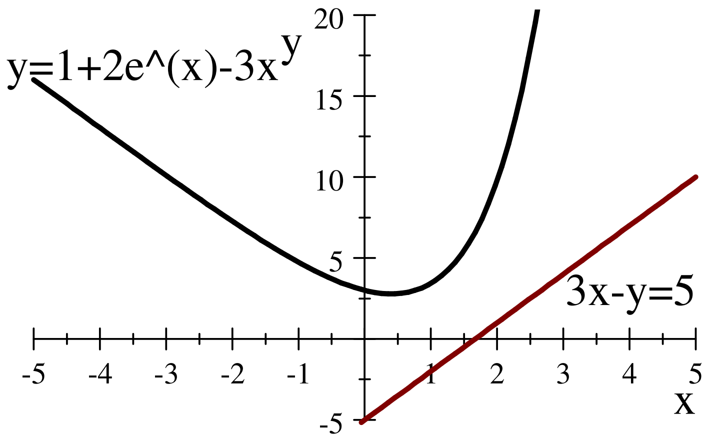
$$f'' = \left(f'\right)' = \left(3.91t^{1.3} + \tfrac{4}{3}t^{-\frac{4}{3}}\right)' = (3.91t^{1.3})' + \left(\tfrac{4}{3}t^{-\frac{4}{3}}\right)'$$

$$f'' = 3.91(t^{1.3})' + \tfrac{4}{3}\left(t^{-\frac{4}{3}}\right)'$$

$$f'' = 3.91(1.3)t^{0.3} + \tfrac{4}{3}(-\tfrac{4}{3})t^{-\frac{7}{3}}$$

$$f'' = 5.083t^{0.3} - \tfrac{16}{9}t^{-\frac{7}{3}}.$$

**Random Problem**. At what point on the curve $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?




$-y = 5 - 3x$

$y = 3x - 5$

Slope is 3

$y' = \frac{d}{dx}(1) + 2\frac{d}{dx}(e^x) - 3\frac{d}{dx}(x)$

$y' = 0 + 2e^x - 3(1) \quad = \quad 2e^x - 3$

$2e^x - 3 = 3, \qquad 2e^x = 6, \qquad e^x = 3$

So, $\ln(e^x) = \ln 3$. Recall that: $\ln(a^x) = x\ln a$, for all $a > 0$.

Therefore: $\ln(e^x) = x = \ln 3 \approx 1.1$.

And this is the $x$ value where the tangent line to the curve is parallel to the line.

**Part b.) Find the tangent line, and Illustrate**.

We know the slope of the tangent line, and an $x$-value, and now we just need a $y$-value in order to complete the point slope form.

$y = 1 + 2e^{\ln 3} - 3(\ln 3) = 1 + 2 \cdot 3 - 3(\ln 3) \approx 3.7.$

So our point is: $(x, y) \approx (1.1, 3.7)$, and our line equation is:

$y - 3.7 = 3(x - 1.1) = 3x - 3.3,$ or

$y = 3x + 0.4.$