

Math 1371 – Lecture 26

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Monday, December 3, 2007

1 Nuts and bolts

1. (re)Read section 6402 Velocity before workshop tomorrow.
2. Office hours this week: MW 11-12, and anytime Th by appt.
3. Exams in the next two weeks:
 - (a) Exam III: Thursday, December 6.
 - (b) Final Exam: **Friday, December 14, 1:30 p.m.** This is listed incorrectly as 12/13 in the course pack.

2 What's happening today: Review

1. Pumping water
2. Velocity: Distance traveled vs. displacement
3. More bucket and chain problems

3 Pumping water

Example 1. A tank is 15 feet long and is full of water. The tank's cross section is an equilateral triangle 10 feet on a side. Find the work required to pump all the water to a level 5 feet above the top of the tank.

4 Distance traveled vs. displacement

If $v(t)$ denotes velocity at time t , then $v(t)$ indicates a direction with its sign: the particle travels in the positive direction when $v(t) > 0$.

From $t = a$ to $t = b$, the *net change in position* or *displacement* of the particle is given by

$$\int_a^b v(t) dt.$$

From $t = a$ to $t = b$, the *total distance traveled* of the particle is given by

$$\int_a^b |v(t)| dt.$$

Example 2. Find the displacement and distance traveled of a particle whose velocity is given by $v(t) = \cos(2\pi t)$, in feet per second, for $0 \leq t \leq 2$ seconds.

Example 3. Find

$$\int_{-5}^5 |4x^3 - 13x + 6| dx.$$

5 Chain and bucket

Example 4. A 150 foot chain that weighs a total of 300 pounds hangs over the side of a bridge. At the end of the chain is a bucket that weighs 30 pounds. The bucket is empty at first, but water is collecting in the bucket at a rate of 0.1 cubic feet for every 10 feet that the bucket is raised.

Find the work required to lift the chain and bucket through the middle 50 feet of the entire distance; that is, from 50 feet from the bottom to 100 feet from the bottom.