Math 1371 – Lecture 27

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1 Nuts and bolts

1. Office hours this week: MW 11-12, and anytime Th by appt.
2. Exams in the next two weeks:
   (a) Exam III: **Tomorrow, in 250 Anderson Hall, 5-6 or 6-7.**
   (b) Final Exam: **Friday, December 14, 1:30 p.m.** This is listed incorrectly as 12/13 in the course pack. (Note: Looking ahead, all sections of this lecture will take the final exam in a different room of Blegen Hall. I will list these room assignments on Monday and Wednesday of next week.

2 What's happening today: More review

1. Bucket and chain
2. Pumping water
3. Volumes of revolution
4. Linear approximation

Example 1. A 150 foot chain that weighs a total of 300 pounds hangs over the side of a bridge. At the end of the chain is a bucket that weighs 30 pounds. The bucket is empty at first, but water is collecting in the bucket at a rate of 0.1 cubic feet for every 10 feet that the bucket is raised.

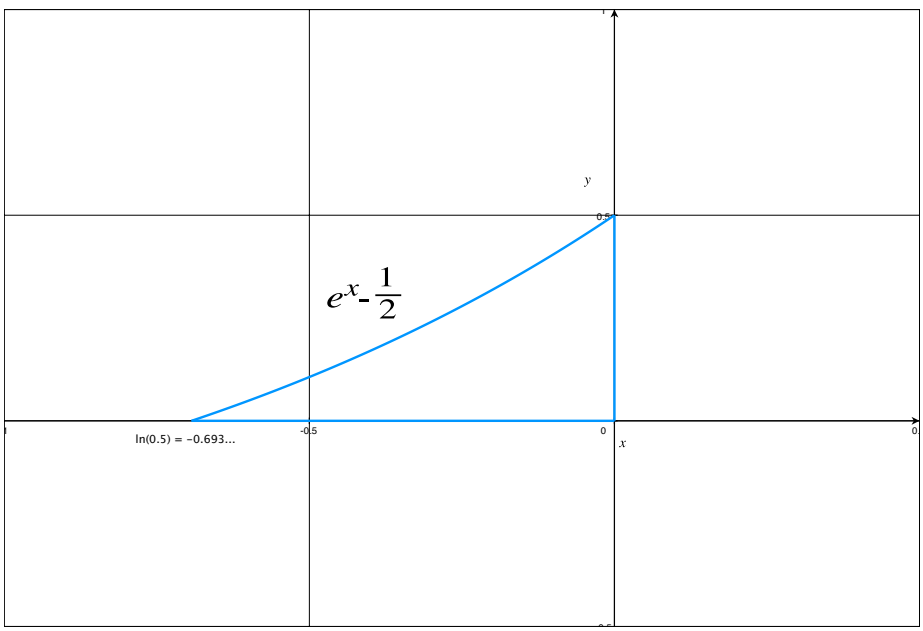
Find the work required to lift the chain and bucket through the middle 50 feet of the entire distance; that is, from 50 feet from the bottom to 100 feet from the bottom.

Example 2. Suppose that a tank is 20 feet long and has a cross section that is a square on top of an isosceles triangle – the square has sides of length 8 feet, and the triangle has (outer) sides of length 5. In other words, it's a "home

plate". Find the work required to pump all the water to the top level of the tank.

Example 3. Let R be the region that is trapped between the graph of $y = e^x - \frac{1}{2}$ and the x- and y-axes.

1. Find the volume of the solid that is formed by rotating R about the line $x = 40$.
2. Find the volume of the solid that is formed by rotating R about the line $y = 40$.
3. Which volume do you expect is larger?



Example 4. First, find the value of the following definite integral using the fundamental theorem of calculus:

$$\int_0^5 (x^2 - 5x)\, dx.$$

Next, find its value DIRECTLY, using a limit of sums. Use the following facts:

$$\sum_{k=1}^{n}(2k-1)^2 = \frac{n}{3}(4n^2-1)$$

and

$$\sum_{k=1}^{n}(2k-1) = n^2.$$

Example 5. Approximate the fifth root of 33.

Added note for example 3:
The volume in part 1 is

$$2\pi(20.5 + 20.5\ln 0.5 - 0.25(\ln 0.5)^2) \approx 38.77$$

and the volume in part 2 is

$$\pi(40.125 + 40.25\ln 0.25) \approx 38.41.$$

Thus, the volume of the solid formed by rotating about the x-axis is slightly bigger.

If you would like to do the calculation for part 1, you might like to use the fact that an antiderivative for xe^x is $xe^x - e^x$.