Math 1371 – Lecture 4

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Monday, September 17, 2007

1 Nuts and bolts

1. Before workshop tomorrow, read the following sections of the text:
   - 6323 The quotient rule
   - 6325 Derivatives of trigonometric functions
   - 6327 Composite functions.
2. Early notice: the first midterm exam is

   Thursday, October 4.

   You have the option to take the test 5-6 p.m. or 6-7 p.m. If you have another class during both of these times, you must tell your workshop leader this week.
3. Office hours this week: MW 11-12, and **Friday 9/21 1-2**.

2 The main point from Lecture 3 on Wednesday

We are developing shortcuts for computing derivatives, such as the power rule and the product rule, but all these shortcuts are derived from the definition of the derivative, and we must remember the graphical interpretation of that definition.

3 What's happening today

1. Exponential functions
2. Quotient rule
3. Trigonometric functions
4. Composite functions

4 Exponential functions

Investigate the following limit numerically.

$$\lim_{n\to\infty}(1+\frac{1}{n})^n$$

n	$(1+\frac{1}{n})^n$
1	2
2	2.25
4	2.4414...
10	2.5937...
100	2.7048...
1000	2.7169...
10000	2.7181...
100000	2.7182...

It turns out that $(1+\frac{1}{n})^n$ tends to a number $\approx 2.71828182846\ldots$, which we define to be the number e.

The function $f(x) = e^x$ is very important in applications, such as interest and population growth.

Here is a very short example:

Example 1. Suppose that you deposit \$1 into the bank. The bank promises 100% interest. At the end of the year, the bank will give back money to you depending on how they decide to compound the interest.

n	$(1+\frac{1}{n})^n$
1	2
2	2.25
12	2.613035...
52	2.692596...
365	2.714567...
365×24	2.718126...

If the interest is compounded continuously, then the bank will give you e dollars at the end of the year.

What does the function $P(t) = e^t$ mean in the context of this application?

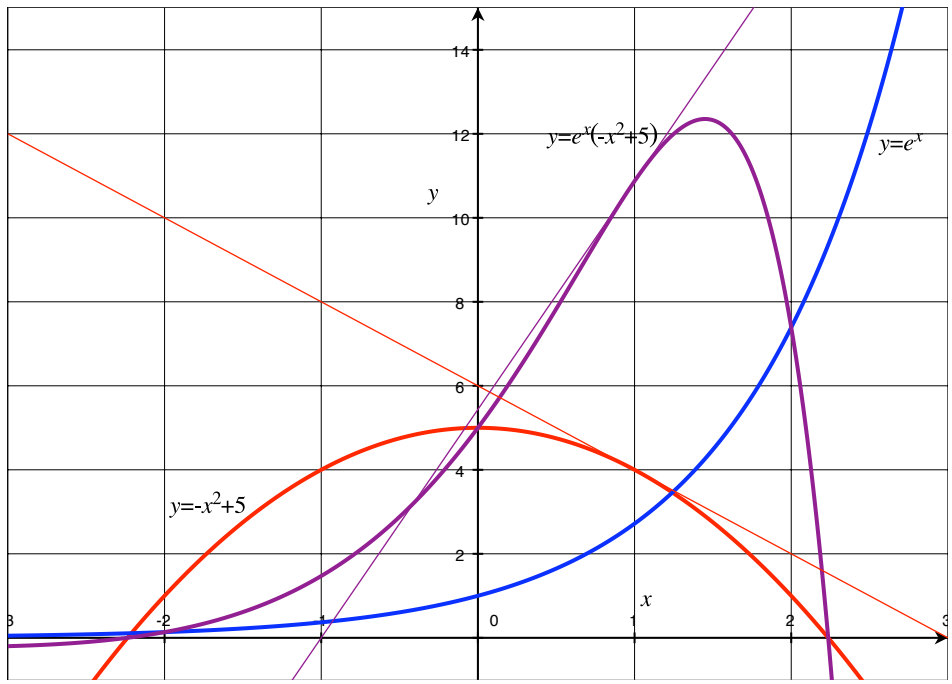
What does the function $P(t) = 3000e^{.05t}$ mean in the context of this application?

For our present purposes, the exponential function is significant because it is the only function whose derivative is equal to itself:

If $f(x) = e^x$, then $f'(x) = e^x$.

Why?

Example 2. Suppose that $g(x)$ is a function with $g(1) = 4$ and $g'(1) = -2$. Let $f(x) = e^x g(x)$. Find $f'(1)$.

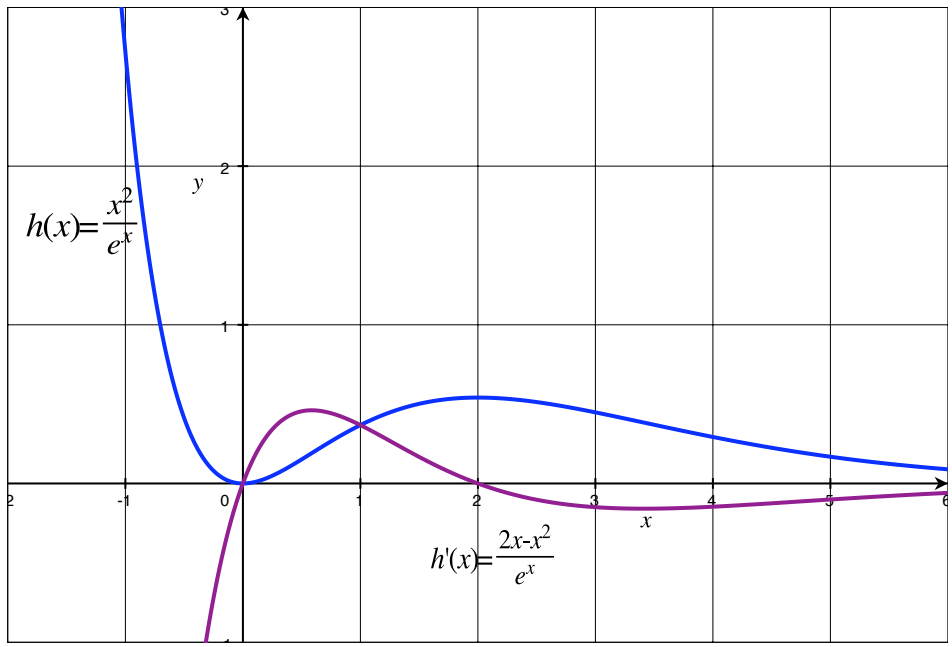


5 The quotient rule

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$$

Example 3. Find the derivative of

$$h(x) = \frac{x^2}{e^x}.$$



Example 4. Suppose $f(x)$ and $g(x)$ are functions with $f(2) = 3$, $f'(2) = -2$, $g(2) = 1$, and $g'(2) = 5$.

Let

$$h(x) = \frac{x^3 g(x)}{e^x f(x)}.$$

Find $h'(2)$.

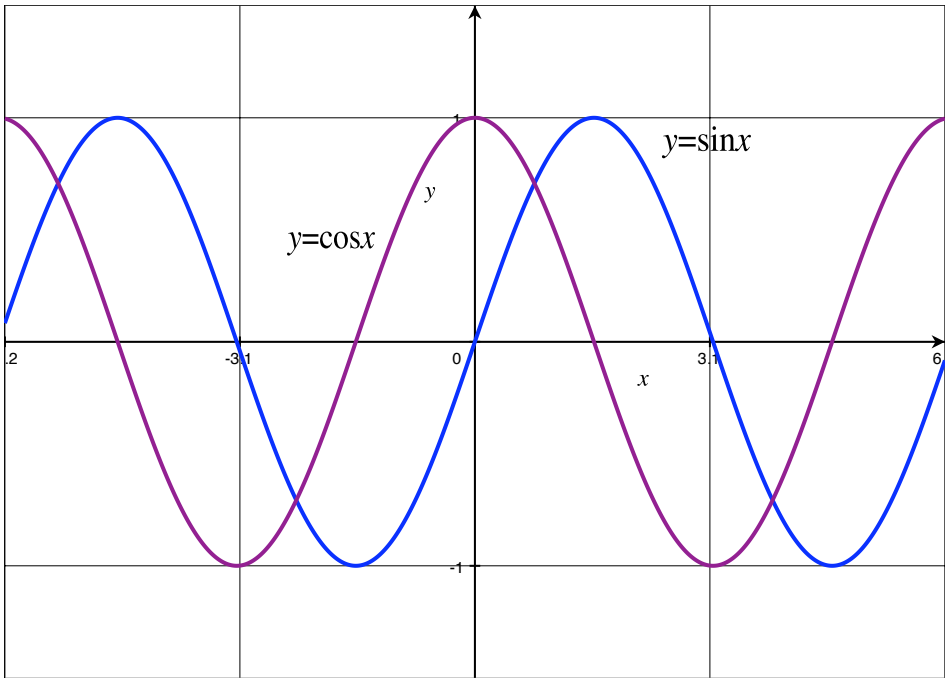
6 Trigonometric functions

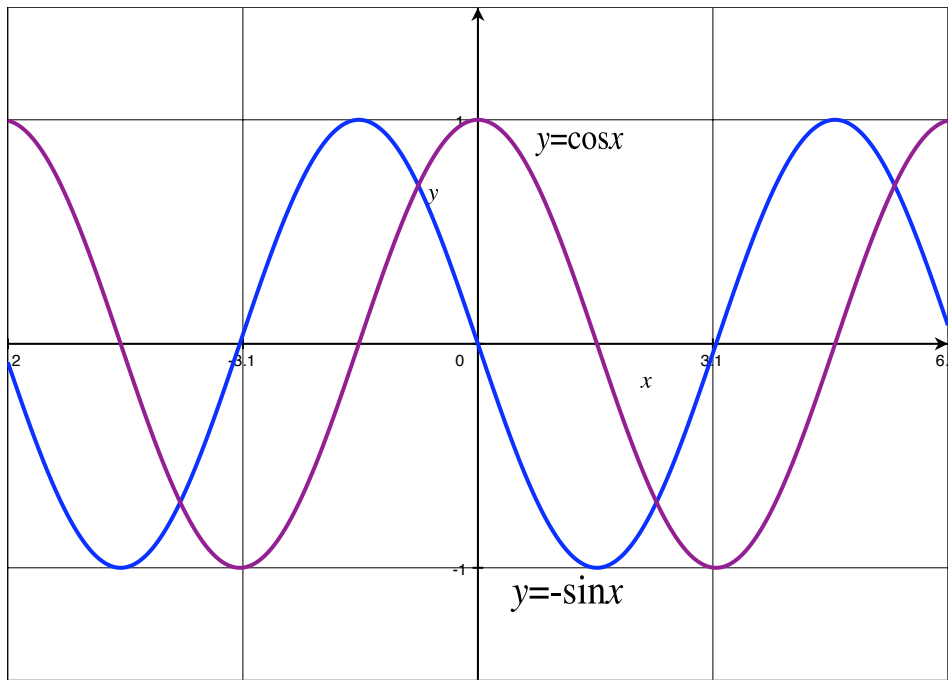
Draw a ray that makes angle x with the positive x-axis. That ray intersects the circle of radius 1 about the origin in one point.

The y-coordinate of that point is called the *sine* of x, denoted $\sin x$.

The x-coordinate of that point is called the *cosine* of x, denoted $\cos x$.

Note: The units of x, the angle, are *radians*. As the ray makes a full revolution, x ranges from 0 to 2π radians. (Think of this as how much circumference of the circle the ray has passed over.)


y=sinx
y=cosx
y
x
1
-1
0
6



7 Derivatives of trigonometric functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

The four other basic trigonometric functions are defined to be *quotients* of $\sin x$ and $\cos x$:

$$\tan x = \frac{\sin x}{\cos x}$$
$$\cot x = \frac{\cos x}{\sin x}$$
$$\sec x = \frac{1}{\cos x}$$
$$\csc x = \frac{1}{\sin x}$$

And hence, their derivatives can be found (not memorized!) by applying the quotient rule.

$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sec x) = \tan x \sec x$$
$$\frac{d}{dx}(\csc x) = -\cot x \csc x$$

8 Composite functions

Suppose you wanted to do the following operations on an input x, in this order:

Take an input x.

Square it.

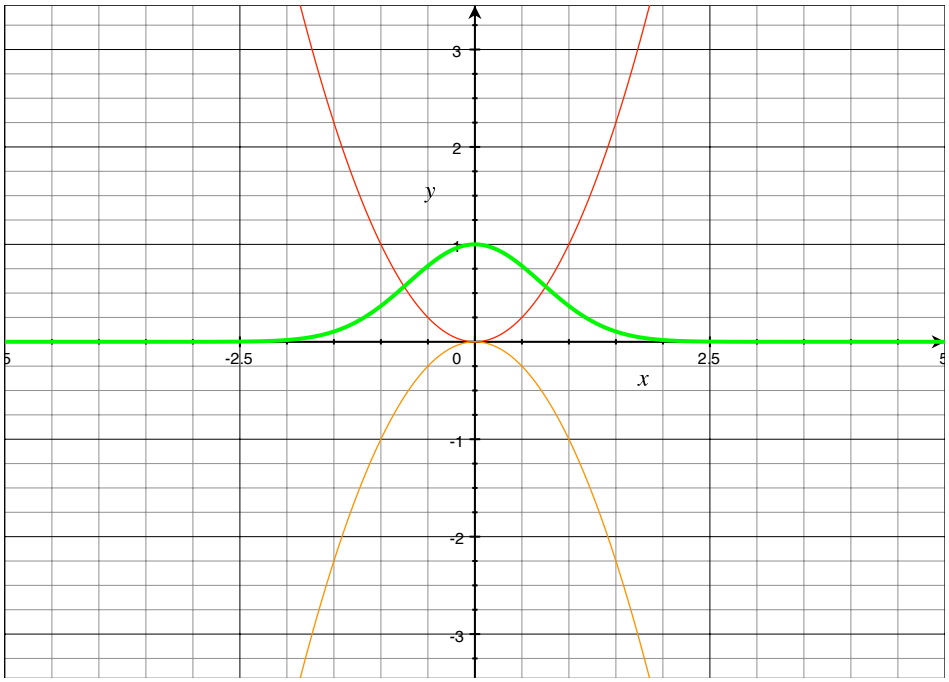
Make the result negative.

Raise e to the power of the result.

We keep track of that with a *composite* function. Let $f(x) = x^2$, $g(x) = -x$, and $h(x) = e^x$.

Then doing all the functions in that order is written

$$h(g(f(x))) = e^{-x^2}.$$


3
2
y
1
-5
-2.5
0
2.5
5
x
-1
-2
-3