1 Nuts and bolts

1. Before workshop tomorrow, read the following sections of the text:
   - 6329 The chain rule
   - 6349 Differentials (Note: This is a few sections ahead in the text. Read with caution, as I will describe later.)

2. Early notice: the first midterm exam is
   **Thursday, October 4.**
   You have the option to take the test 5-6 p.m. or 6-7 p.m. If you have another class during both of these times, you must tell your workshop leader this week.

3. Office hours this week: MW 11-12, and **Friday 9/21 1-2.**


2 The main point from Lecture 4 on Monday

We continue to expand our library of functions (exponential, trigonometric, etc.), and we continue to develop shortcuts for computing their derivatives, keeping in mind the numerical and graphical meaning of those calculations.

3 What’s happening today

1. The chain rule – this is how we find derivatives of composite functions.

2. Differentials – the language of linear approximation
4 Review: trigonometric functions

Remember that last time we recorded, after gathering graphical evidence, that
\[ \frac{d}{dx}(\sin x) = \cos x \]
and
\[ \frac{d}{dx}(\cos x) = -\sin x. \]

Also, we defined the other four basic trigonometric functions, tan, cot, sec and csc, as quotients of sin and cos. Hence we can find their derivatives by applying the quotient rule.

**Example 1.** Find the derivative of \( \cot x \).

5 Review: composite functions

Let’s return to our example at the end of last time: \( e^{-x^2} \).

![Graph of \( y = e^{-x^2} \)]
6 The chain rule

\[
\frac{d}{dx}(g(f(x))) = g'(f(x)) \cdot f'(x).
\]

“The derivative of a composite function is
the derivative of the outside function evaluated at the inside function
times
the derivative of the inside function.”

**Example 2.** Now let’s think of $e^{-x^2}$ as a composite of two functions: write $g(f(x)) = e^{-x^2}$, where $f(x) = -x^2$ and $g(x) = e^x$. Find

\[
\frac{d}{dx}(e^{-x^2}).
\]
Example 3. Let \( f(x) = \sin(x + 0.1\sin(20x)) \). Find \( f'(x) \).

7 Differentials

... help us keep track of how well a tangent line to the graph of a function is approximating that function.

Keep in mind the following picture:
If $x$ changes by this much...
then the function changes by this much,
and we call that the \textsc{Increment}.

\[
\Delta y = f(x + \Delta x) - f(x)
\]
To summarize the notation and terminology we used in the picture:

For a function \( f(x) \) at a point \( x \):
Suppose we change \( x \) by a little bit \( \Delta x \). Then how much the function actually changes is given by the increment:

\[
\Delta y = f(x + \Delta x) - f(x).
\]

Suppose we change \( x \) by a little bit \( dx \). Then the tangent line approximates that the function changes by this much, which is called the differential:

\[
dy = f'(x)dx.
\]

**Example 4.** For \( f(x) = e^x \), find the increment and the differential. Compare their values when \( x = 0 \) and \( \Delta x = dx = 0.1 \).

The notation of increments and differentials gives us a new but equivalent way to write the definition of derivative:

\[
\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{dy}{dx}.
\]

**Example 5.** You have a spherical balloon of radius 10 cm. Use a differential to estimate how much more surface area the balloon has when you blow up the balloon further so that its radius has increased by 1 cm.