Math 2263.002  
Summer 2009  
Exam 1 solutions

The exam is worth 150 points. Thirty-six students took the exam. The mean was 107.4 (71.6%), and the median was 114.5 (76.3%).

1. \( \mathbf{n}_1 = (1, -1, 2) \) (5 points). \( \mathbf{n}_2 = (-1, 0, 1) \) (5). \( \mathbf{n}_2 \times \mathbf{n}_1 = (1, 3, 1) \) (5). The point \((2, -1, 0)\) is on the line of intersection (5). \( \mathbf{r}(t) = (2, -1, 0) + t(1, 3, 1) \). \( x(t) = 2 + t, \ y(t) = -1 + 3t, \) and \( z(t) = t \) (5).

2. Along \( y = 0 \), \( \frac{5x^2}{2x + y} = \frac{9}{x} = 0 \), when \( x \neq 0 \) (5). Along \( y = x \), \( \frac{5x^2}{2x + y} = \frac{5x^3}{2x} = \frac{5}{2} \), when \( (x, y) \neq (0, 0) \) (5). Thus, the limit does not exist. (5).

3. (a) \( f(1, -2) = 1 \). \( f_x = 6x - 2xy^2 + 2 \) (3). \( f_x(1, -2) = 0 \) (3). \( f_y = -2x^2y \) (3). \( f_y(1, -2) = 4 \) (3). An equation for the tangent plane is \( z - 1 = 4(y + 2) \) (3).
   
   (b) \( \mathbf{n} = (0, 4, -1) \) (5). An equation is \( z = 4y \) (5).

4. (a) \( \partial f \partial x = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \partial y \partial x \) (3) = \( 2x \sin y + y^2 e^{xy})(2) + (x^2 \cos y + e^{xy} + xye^{xy})(s) \) (3 for each of 4 factors).
   
   (b) \( (x, y) = (4, 2) \) (2). \( \frac{\partial f}{\partial x}(2, 1) = (2)(8 \sin 2 + 4e^2)(3) = (16 \cos 2 + e^2 + 8e^2) \) (4 for each of 2 summands) = \( 16 \sin 2 + 26e^2 + 32 \cos 2 \).

5. (a) \( \nabla f = \langle x^2 - 2, -2x^2 - 2xy, -x^2 + 3z^2 \rangle \) (2 for each entry).
   
   (b) \( \nabla f(-2, 3, -1) = (1, -1, -10) \) (2). \( \mathbf{u} = \frac{1}{\sqrt{14}} (2, 1, 3) \) (2). \( D_u f(-2, 3, -1) = \nabla f(-2, 3, -1) \cdot \mathbf{u} (2) = (1, -1, -10) \cdot \frac{1}{\sqrt{14}} (2, 1, 3) = \frac{-29}{\sqrt{14}} \) (4).
   
   (c) The maximum rate of change is \( |\nabla f(-2, 3, -1)| = \sqrt{102} \) (6), and it occurs in the direction of the unit vector \( \frac{1}{\sqrt{102}} \) (1, -1, -10) (3).

6. (a) \( f_x = 3y - 2xy - y^2 \) and \( f_y = 3x - x^2 - 2xy \) (2 each). There are four critical points: \((0, 0), (3, 0), \) and \((0, 3)\) are all saddle points, and there is a local max at \((1, 1)\). (2 for each CP and 2 for each classification).
   
   (b) \( f = 0 \) along the entire boundary, including the hypotenuse (5). That is the minimum value (5), and the maximum is \( f(1, 1) = 1 \) (5).