In all cases where numerical answers are required, give exact answers, not decimal approximations.

1. (24 points; 3 each)
   (a) Answer each question with a short phrase.
      i. What does it mean for a curve $C$ to be closed?
      
      ii. What does it mean for a closed curve $C$ to be simple?
      
      iii. What does it mean for a simple, closed curve $C$ to be positively-oriented?
      
      iv. What does it mean for a vector field $\mathbf{F}$ to be conservative?
      
   (b) Let $\mathbf{F} = \langle y + 2xz, x + 3y^2z, x^2 + y^3 \rangle$.
      i. Find curl $\mathbf{F}$.
      
      ii. Find div $\mathbf{F}$.
      
      iii. Is the vector field $\mathbf{F}$ conservative? Give a short justification.
      
      iv. Is there a vector field $\mathbf{G}$ such that $\mathbf{F} = \text{curl } \mathbf{G}$? Give a short justification.
2. (26) Let $\mathbf{F} = \langle y + e^{x^2}, x^2 + e^{y^2} \rangle$, and let $C$ be the positively-oriented boundary of the triangle that has vertices $(1, 1)$, $(3, 1)$, and $(1, 3)$. Find $\int_C \mathbf{F} \cdot \mathbf{dr}$. 
3. (25) Find the mass of the solid that lies in the first octant, is trapped between the spheres

\[ x^2 + y^2 + z^2 = 16 \]

and

\[ x^2 + y^2 + z^2 = 25, \]

and lies above the top half of the cone \( 3z^2 = x^2 + y^2 \) if the density of the solid, in grams per cubic centimeter, is given at each point by the distance between the point and the origin.
4. (25) Let \( \mathbf{F} = (-z \sin x \sin y, z \cos x \cos y, \cos x \sin y) \), and let \( C \) be the curve from \((0, 3, 5)\) to \((0, -3, 5)\) that traces a semi-circle in the \( yz \)-plane given by \( y^2 + (z - 5)^2 = 9, \ z \geq 5 \). Find

\[
\int_C \mathbf{F} \cdot d\mathbf{r}.
\]
5. (25) Let
\[ F = \langle z, e^{\cos(z)^2}, xy \rangle, \]
and let \( C \) be the directed line segment from \((2, 1, -3)\) to \((-1, 1, 4)\). Find
\[ \int_C F \cdot dr. \]
6. (25) Find the mass of a wire represented by the graph of the function $y = x^3$ between the points $(1,1)$ and $(3,27)$, if the density of the wire, in grams per centimeter, at each point is given by the cube of the distance from the point to the $y$-axis.