MATH 2373
IT Linear algebra and differential equations
Spring 2009

Lecture 020: MW 2:30 - 3:20, 131 Tate Lab

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Workshop and lab sections:

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Text: Course packs by Chester Miracle. Buy the first one as soon as possible from Alpha Print, 1407 4th St. SE, in Dinkytown.

Information pertaining to all lectures and workshop sections of Math 2373 is in the front of the first course pack, including information about grading and guidelines for workshop participation.

Your responsibilities

• Read the relevant textbook section(s) in advance of lecture.

• Participate in the group work in workshop and in the labs.

• Try assigned homework well in advance.

• Seek help if you need it – office hours, send email, tutoring hours in Lind Hall.

My responsibilities here

• Provide context and motivation.

• Relate theory and practice.

• Model precise mathematical communication.

• Link lecture content with workshop and lab practice.
Overview of the course

Our general goals are:

• to improve our critical thinking skills, and

• to improve our mathematical communication, written and spoken.

Today: Gaussian elimination

Why would we care to study that, or, for that matter, any of linear algebra or differential equations?

What is linear algebra?

Why do we care about linear algebra?

What are differential equations?

Why would we study linear algebra and differential equations in the same course?

Now, on to some linear algebra:

Example 1. Solve the following system of linear equations:

\[ \begin{align*}
2x + 3y &= 16 \\
5x - 2y &= 21
\end{align*} \]

We want to replace an augmented matrix with another augmented matrix that represents a system WITH THE SAME SOLUTIONS.

We do that using elementary row operations, and that process is called Gaussian elimination.

Elementary row operations

• Interchange any two rows

• Multiply a row by a nonzero constant.

• Replace a row with the sum of that row and a constant multiple of another row.

Our goal is to produce an augmented matrix that is in row-echelon form:

• Rows of all zeros is at the bottom.

• The first nonzero entry of every nonzero row is a 1.

• Those leading 1’s “go down and to the right”.
Back to **Example 1.** Solve the following system of linear equations, this time using Gaussian elimination:

\[
\begin{align*}
2x + 3y &= 16 \\
5x - 2y &= 21
\end{align*}
\]

**Example 2.** Solve the following system of 3 linear equations in 3 variables.

\[
\begin{align*}
2x + 6y + z &= 7 \\
x + 2y - z &= -1 \\
5x + 7y - 4z &= 9
\end{align*}
\]
Example 3. Solve the following system of 3 linear equations in 3 variables.

\[
x + 3y - 2z = -7
\]
\[
4x + y + 3z = 5
\]
\[
2x - 5y + 7z = 19
\]

\[
\begin{bmatrix}
1 & 3 & -1 & -1 \\
0 & 2 & 3 & 9 \\
0 & -3 & 1 & 14
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & -1 & -1 \\
0 & 1 & 2 & -9 \\
0 & -3 & 1 & 14
\end{bmatrix}
\]

\[
\frac{1}{2}R_2
\]

\[
\begin{bmatrix}
1 & 2 & -1 & -1 \\
0 & 1 & 2 & -9 \\
0 & 0 & -3 & 1
\end{bmatrix}
\]

\[
3R_2 + R_3
\]

\[
\begin{bmatrix}
1 & 2 & -1 & -1 \\
0 & 1 & 2 & -9 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

This matrix is in row-echelon form. Back-substitute.

Solution: \(x = 10, y = -3, z = 5\)
This matrix is in row-echelon form. There is no restriction on $z$!

**Solution:** The set of solutions can be described as follows:

\begin{align*}
  x &= 2 - t, \\
  y &= t - 3, \\
  z &= t,
\end{align*}

for any real number $t$. There are infinitely many solutions.