Matrix addition

If $A$ and $B$ are matrices, then $A + B$ is defined only if the dimensions (the numbers of row and columns) of the matrices are the same. In that case, $A + B$ has the same dimensions and is defined by entry-wise addition.

Example 1. Let

$$A = \begin{bmatrix} 5 & -1 \\ 2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 \\ 1 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -1 & 0 \\ 2 & 3 & -1 \end{bmatrix}.$$ 

Find $A + B$, $B + C$, $B + A$, if possible.

Scalar multiplication

If $A = [a_{ij}]$ is any matrix, and $k$ is any real number, then we define $kA = [ka_{ij}]$.

Example 2. If $A = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $I$ is the $2 \times 2$ identity matrix, write the matrix $A - \lambda I$.

(cf. 7666 Eigenvalues)

Example 5. If $A$ is any $m \times n$ matrix, find $AI$ and $IA$.

Matrix multiplication

First, recall the dot product:

Example 3. Let $v = (3, -2, -1)$ and $w = (1, 2, -1)$. Find $v \cdot w$.

If $A$ and $B$ are matrices, then their product $AB$ is defined only if the number of columns of $A$ equals the number of rows of $B$.

In that case, if $A$ is an $m \times n$ matrix and $B$ is an $n \times p$ matrix, then $AB$ is the $m \times p$ matrix whose $ij$-entry is the dot product of the $i$th row of $A$ and the $j$th column of $B$.

Example 6. Write the following system of linear equations as a matrix equation.

$$x_1 + 2x_2 = 5,$$
$$3x_1 - 2x_2 = -2.$$
First order linear differential equations have the form
\[ a(x) \frac{dy}{dx} + h(x)y = c(x). \]

**Example 7.** Solve the differential equation
\[ xy' - 3y = x^2, \]
for \( x > 0 \).

Exponential growth and decay