MATH 2373.020 – Lecture 3 – 1/28/09

HW2, due Tuesday, February 3:
7586 #1-6
7588 #1-7
7590 #1-7
7552 #1-4
7555 #1-3
7560 #1-4

Find lectures slides, M-files, and now podcasts at:
`http://www.math.umn.edu/~mosher/math2373`

Calendar

1/21	7584 Gaussian elimination
1/26	7586 Determinants 7588 Cramer's rule 7552 Separable differential equations
Today:	7590 Matrices 7555 First order linear differential equations 7560 Exponential growth and Newton's law of cooling

Operations on matrices

How do we add two matrices?
How do we multiply a matrix by a scalar?
How do we multiply two matrices?

And then: what is the significance of those operations?

Matrix addition

If A and B are matrices, then $A+B$ is defined only if the dimensions (the numbers of row and columns) of the matrices are the same. In that case, $A+B$ has the same dimensions and is defined by entry-wise addition.

Example 1. Let

$$A = \begin{bmatrix} 5 & -1 \\ 2 & 3 \\ -2 & -2 \end{bmatrix}, B = \begin{bmatrix} -3 & 0 \\ 1 & -3 \\ 4 & 5 \end{bmatrix}, C = \begin{bmatrix} 5 & -1 & 0 \\ 2 & 3 & -1 \end{bmatrix}.$$

Find $A+B$, $B+C$, $B+A$, if possible.

Scalar multiplication

If $A = [a_{ij}]$ is any matrix, and k is any real number, then we define $kA = [ka_{ij}]$.

Example 2. If $A = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$ and I is the 2×2 identity matrix, write the matrix

$$A - \lambda I.$$

(cf. 7656 Eigenvalues)

Matrix multiplication

First, recall the *dot product*:

Example 3. Let $\mathbf{v} = \langle 3, -2, -1 \rangle$ and $\mathbf{w} = \langle 1, 2, -1 \rangle$. Find $\mathbf{v} \cdot \mathbf{w}$.

If A and B are matrices, then their product AB is defined only if the number of columns of A equals the number of rows of B.

In that case, if A is an $m \times n$ matrix and B is an $n \times p$ matrix, then AB is the $m \times p$ matrix whose ij-entry is the dot product of the ith row of A and the jth column of B.

Example 4. Let

$$A = \begin{bmatrix} 5 & -1 \\ 3 & -2 \\ 2 & -3 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & -1 & 0 \\ 3 & 3 & 1 \\ 1 & 7 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$

Find AB and BA if possible.

Example 5. If A is any $m \times n$ matrix, find AI and IA.

Example 6. Write the following system of linear equations as a matrix equation.

$$x_1 + 2x_2 = 5,$$

$$3x_1 - 7x_2 = -2.$$

First order linear differential equations have the form

$$a(x)\frac{dy}{dx} + b(x)y = c(x).$$

Example 7. Solve the differential equation

$$xy' - 3y = x^2,$$

for $x > 0$.

Exponential growth and decay