MATH 2373.020 – Lecture 9 – 2/18/09

HW5, due tomorrow:
7555 #9
7572 #1,2

Exam 1 tomorrow: Tate Lab 150,
5-6 or 6:05 - 7:05

Office hours: tomorrow 1 - 2:15

Today:

- coefficient homogeneous DE

- first-order linear DE *with constant coefficients* (as arises from circuit problems)

- linear combinations of vectors

- matrix inverse using Gaussian elimination

- equilibrium solutions of a logistic DE

Example 1. Solve the initial value problem

$$\frac{dy}{dx} = \frac{4x+3y}{x+2y},$$

$$y(1) = 1.$$

from Monday (example 3): $y(1.1) \approx 1.2\bar{3}$, an overestimate since $y'' < 0$ near $x = 1$.

Example 2. Find the general solution of the differential equation

$$y' + 2y = \sin t.$$

This is a linear first-order DE with *constant coefficients*.

- first, using an integrating factor

- second, using the theorem that says: if you know ONE particular solution, then the general solution is the sum of that particular solution and the general solution of $y' + 2y = 0$. Note: this only applies when there are constant coefficients on y and y'.

Example 3. Are the following vectors linearly independent?

$$\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}2\\0\\0\\-1\end{pmatrix}, \begin{pmatrix}-3\\1\\1\\3\end{pmatrix}$$

Example 4. Find the inverse of the matrix

$$\begin{bmatrix}1 & 2 & 1\\2 & 1 & 1\\1 & 1 & 2\end{bmatrix}.$$

-first, using Gaussian elimination

-second, using cofactors: The inverse is the transpose of the matrix of cofactors divided by the determinant.

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2 - 2R_1, R_3 - R_1 \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -3 & -1 & -2 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

$$-3R_3 + R_2 \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & -4 & 1 & 1 & -3 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

$$-R_3, -\frac{1}{4}R_2, R_{23} \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

$$R_3 + R_2, -R_3 + R_1 \begin{bmatrix} 1 & 2 & 0 & \frac{5}{4} & \frac{1}{4} & -\frac{3}{4} \\ 0 & 1 & 0 & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

$$-2R_2 + R_1 \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

The inverse matrix is

$$\begin{bmatrix} -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix}.$$

Example 5. Find the general solution of the differential equation $y' = 1 - y^2$, find all equilibrium solutions and determine which are stable.